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Dynamic Systems Approaches and Modeling of Developmental Processes

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The notion of development plays an important role in cultural, political and personal discourse. For instance, in the newspaper, we read about the problems of developing countries. At another level, personal development is considered an important issue and people spend considerable effort in promoting their development and that of otheres for whom they have responsibility.

The notion of development has subtle and diverse meanings, which are reasonably adequately covered by everyday language. Scientific discourse about development as such, apart from the technical aspects of applying it to one or other specific domain of inquiry, is not much better developed, if at all, than its everyday counterpart. It might be interesting, therefore, to look at the intuitive meaning of the notion of development, in an attempt to uncover aspects that might be worth considering in more formal, scientific approaches. One way to do so is to look at the original meaning of the word 'development', its etymology.

The Notion of Development: an Approach Through its Etymology

The English word 'development' stems from the Old French *desvoloper*, which means 'to unwrap'. The German and Dutch words *Entwicklung* and *ontwikkeling* are literal translations of that term. In its historical roots, the word 'development' is related to the Latin *evolutio* (to unroll) and the semantically related word *explicatio* (to unfold). The Latin words referred among others to the unfolding or unrolling of book rolls. The latter meaning is still preserved in the word 'explain', which therefore bears an unexpected relationship to the word 'development' (Thomae, 1959; Trautner, 1978).

There is a lot of metaphorical connotation in these semantic forebears of the notion of development that is still preserved in its current use (for a more extensive treatment of this issue, see van Geert 1986a; 1986b; 1988; 1990). The notions of unwrapping and unfolding carry a meaning of something that is inside the wrapping and that is taken out. Another metaphoric meaning contained in those terms is that of a folded structure that is folded out, similar to rosebuds whose petals grow and meanwhile fold out to bring forth the rose's mature shape. The unfolding is a particularly nice metaphor, since it suggests that the form is already there at the beginning in some germinal state and that it is reached in a series of qualitatively different intermediate forms that correspond with each step of the unfolding. Many years ago, Nagel (1957, p. 17) probably hinted at this metaphor when he described development as what happens to a system with a specific structure and initial capacities, characterized by a series of successive changes leading to relatively permanent, new structural properties.

The notion of development as used in colloquial discourse – and scientific discourse too, for that matter – carries the meaning of an internally driven force acting in a specific context. For instance, we speak about developing a photograph or developing a piece of land. The body develops because the person consumes food and exercises its growing capacities; people develop their skills by learning from others and using the skills where appropriate. We speak about the development of logical thinking, of language and so forth. Although we are well aware of the fact that such things will hardly ever develop out of their proper contexts, we also assume that they change because of some internal drive or process. All this is reminiscent of Bergson's notion of the *élan vital*, the vital drive, that governed the processes of life and evolution, at least as was thought around the turn of the nineteenth into the twentieth century. As far as development is concerned, there exists a non-specific relationship between the context and the process of development that takes place in this context (the context is needed, but it does not prescribe or prefigure the development). This is very different from what is supposed to happen in a process of learning, or of cultural transmission, where the environment directly governs – or at least attempts to do so – the internal changes in the learning person (what is learned is what is given or transmitted: van Geert, 1986a).

In addition to this aspect of an internal drive, development has a connotation of progress, of increasing complexity, structure and order. One does not develop to become less or worse. This idea of intrinsic progression is also entailed in theories of development, or more particularly, theoretical models that described the course of the developmental process. Based on the structure of those models, we can make a distinction between retrospective theories and prospective theories. Retrospective theories are those that look at the developmental process from the perspective of an end state and view all preceding states in light of this end state. Under this perspective, the developmental process is like a logically necessary move towards the preset end state. Prospective theories are those that look at development from the perspective of its initial state and the mechanisms that operate on that initial state. They see development as a fundamentally open process (for a more detailed analysis, see van Geert 1987a; 1987b; 1987c; 1987d; 1988). It is interesting to note, however, that the concept of development sometimes entails a notion or progress that creates its own progress criteria. For instance, when we speak about the development of a new artistic style or a new form of philosophical thought, that new form or style sets at least part of the criteria by which it has to be judged. In fact, one can almost distinguish two kinds of approaches to the issue of progress as it relates to development. One is, so to say, more conservative, in that it sees progress as the reaching of some preset standard or criterion (like a person with a well-developed taste, implying that his taste meets some accepted, culturally valued criterion). The other is a more progressive approach, which focuses on the fact that new criteria for judging that progress emerge simultaneously with the progress itself. This latter approach to development emphasizes the aspect of novelty, of development as the creator of something new. Note that, since development is assumed to be internally driven, novelty does not necessarily imply uniqueness. It is conceivable that each and every newborn child develops through a series of structural possibilities – such as the Piagetian stages, for instance – which are entirely new from the child's standpoint, which have neither been transmitted nor been genetically coded, but are nevertheless (almost) similar for all children.

In summary, the notion of development as it features in colloquial speech and general models alike entails a certain tension between opposing traits. Development entails an aspect of pre-destination – something unfolds that is already there – but also an aspect of coming-into-being that is more than the simple uncovering of what is there already at the beginning. Second, development involves an aspect of self-governed, internally driven change, but also an aspect of context dependency, of the necessity of an external support. It also involves the idea of increasing order and structure, a progress towards higher quality and even the creation of new forms and structures. In summary, development is a highly particular process that cannot and should not be reduced to a simple causal process driven by either internal or external conditions. Its particular nature finds its expression in the rather ambiguous, somewhat unclear nature of the concept as it is used in different forms of discourse.

Scientific Attacks on the Notion of Development

One of the main problems with the notion of development as described in the preceding section is that there exists a lot of common sense evidence for it, but very little real theory to back it up. For instance, parents with some experience of raising children find that educating children requires a lot of effort, while on the other hand, the child's growing up has a logic of its own and is all but directly governed by the parents' goals and actions. Every new organism comes into existence through a somatic developmental process that transforms more or less non-specific energy supplied from outside, in the form of food, for instance, into a highly specific body form. Historically, cultures have changed and developed without a mastermind that governed their paths (wherever such a mastermind took the lead, things went – often dramatically – wrong).

The explanation of development has long since been the endeavor of philosophers more than of natural scientists. Kant (1724-1804), for instance, viewed the organism as a whole of interdependent components and aspects, sustained by its inherent logic. In the late nineteenth and early twentieth centuries, philosophers and historians tried to explain cultures as developing wholes, with their own internal drives and their own life span and developmental stages (see for instance the work of Spengler, 1880-1936, on the decline of western civilization). We have already encountered the French philosopher Bergson (1859-1941), for instance, who conceived of an *élan vital*, which is a vital impulse that governed the unfolding of life's inherent tendencies.

In stark contrast to these mostly philosophical attempts at explaining and describing the process and notion of development, the natural sciences – at least since Newton – have gradually shifted away from the core meanings of development and have become increasingly critical about it.

The second law of thermodynamics, which is a fundamental law of nature, deals with the fate of order and structure in the universe. It says that order can never spontaneously increase. It must decay unless it is driven by some external source, which must have a higher level of order than the order it is able to create. The history of the discovery of the second law of thermodynamics is intimately related to the emergence of industrial society and the massive use of machines (Atkins, 1984). Machines need energy to accomplish something and what they accomplish is always less than the net energy that has been put into them. There exists an analogy to this process of heat transmission in the transmission of information through a channel: there is always more information in the sender than in the receiver, since the transmission through the channel leads to an irreversible and inevitable loss of information. Since the laws of thermodynamics also govern animate nature, development – if viewed as a spontaneous increase of order – must be an illusion.

Not only in physics, but also in biology, the idea of development as an inherent trend towards improvement came under severe attack. Historically, the idea of gradual improvement of successive life forms became known as the *Scalae Naturae*, the ladder of nature or the Great Chain of Being. The idea was that life begins (in a non-historical sense of that word, however) with the most primitive organisms and advances through stages of increasing complexity up to the most complex of them all, man. Although this notion did not entail a concept of time – and thus of evolution or development as we see it today – it did entail a progression towards increasing complexity. The idea was vindicated by the so-called Rational Morphologists, who saw the form of the organisms' bodies as coherent wholes and the relation between the body forms of species as one of an underlying structural logic across the species' boundaries. These concepts were wiped out completely with the advent of Darwinian evolutionary theory. Darwin made an important contribution in that he introduced the notion of time as an inherent factor in the explanation of the forms and properties of biological species and in doing so he introduced the notion of phylogenetic change in addition to the already familiar notion of ontogenetic change (the growth of a single organism). Meanwhile, Darwin's theory of evolution discarded all reference to a notion of development, of progress-directed deployment of inherent structure. The major mechanism is that of selection of accidental variations by an environment that selectively favors some variations over others. Selection leads to increasingly better adaptations of the species to its environment, but this result is not due to the working of some inherent tendency towards betterment. Evolution does not necessarily lead to increase of structure and complexity. If survival is better warranted by loss of complexity and structure than by gain, then loss of complexity is what occurs. Thus, whatever survives is better adapted than anything that does not survive and in that sense the predicate 'fittest' can only be given after the facts (after the selection has taken place). However, if evolution is looked at from a retrospective point of view – that is, given the present state of affairs at the stage of biological species – it seems as if evolution was indeed driven to some highly complex end state, the complex tree of life that we witness today. But this retrospective look is highly deceptive. We should realize that the state of species evolution as we know it today is a highly coincidental matter. The stage could just as well have been populated in a dramatically different way and what we call increasing complexity is nothing but the result of the fact that the only direction evolution could go was to increase complexity in some species, whereas the most successful species are still very close to the 'simplicity' of early life forms (both arguments are strongly defended in Gould, 1989; 1996). The principle of selection is entirely dependent on the mechanism of variation, because if there is no variation there are no differences and if there are no differences it doesn't matter what is selected, because the result will remain the same. Thus, understanding the source and mechanisms of variation is of crucial importance to understanding the course of evolution. Variation is something that applies to the form and properties of the organism, i.e. their morphology, and this morphology is the product of the mechanisms of morphogenesis.

The process of morphogenesis – the growth of a single organism – remained largely a mystery until the discovery of genetics, especially the atomistic approach originally developed by Mendel. It explained the growth of the organism as the result of building instructions contained in the genes. This approach to genetics basically pays tribute to the second law of thermodynamics: the complexity of the developing body is entirely entailed in its genetic starting point. Form and order do not emerge spontaneously but are inscribed in the genetic instruction book. Note that this approach to genetics agrees very well with the notion of development as the unfolding of what is already there, though concealed in the organism's deepest, genetic kernel. However, it differs from the more naive developmental view in that it acknowledges that the end result of morphogenesis is, structurally speaking, nothing more than what was already contained in the genetic instruction. The mechanisms of morphological variation are thus reducible to those of genetic variation, i.e. mutations. In summary, the biological view on the evolution of species – their phylogenetic development, so to speak – seems to be reducible to the principle of instruction sets (the genome), random variation of this genome and selection of the most favorable expressions of the potential genomes. (I say 'seems to be reducible', since I shall argue that modern views on morphogenesis take a somewhat different approach; by and large, however, even today the widespread view on the evolution of species is still very similar to the one just sketched.)

As to developmental psychology, the notion of development has had its strong defenders in scholars that did their main work in the first half of the twentieth century (Piaget, Werner, Vygotsky and others). Later scientific developments, however, gradually moved the field away from its concern with development proper (van Geert, 1998c). One is the adoption of linear statistical modeling, which no doubt increased the methodological rigor of research, but also replaced the notions of wholeness and mutuality characteristic of the older conceptualizations of development with one of asymmetric relationships between variables. As a result, developmental psychology gradually turned into a study of group differences, the groups defined by their ages. A second change in the field had to do with the emergence of a new approach to the study of language, namely Chomskyan linguistics, which was strongly inspired by a centuries-old rationalism. With regard to language development, Chomsky showed that language – *qua* human knowledge – is underdetermined by the input, that is, the language addressed to a language-learning child. That is to say, it is logically impossible to extract the grammar of a language on the basis of the linguistic environmental input alone. Nevertheless, children do acquire the grammar of their language and they do so easily and rapidly. Since the grammar is not transmitted by giving linguistic input, it follows that knowledge of the grammar must be present in the language learner in advance. Language development is therefore basically the unfolding of innately present knowledge, with the innate knowledge actualized in the form of some specific language. This view of language acquisition is highly reminiscent of genetic information transmission as conceived of in the atomistic, Mendelean view. In the 1970s, this view of language acquisition was highly applauded by a group of biologists and geneticists who gathered at the Abbaye de Royaumont to witness a discussion between Piaget and Chomsky. The discussion, which was laid down in a widely cited book (Piattelli-Palmarini, 1980), led to a victory, if one may use that word, of the Chomskyan view and to the defeat of Piaget's developmentalism. Piaget's view was identified with an obsolete vision of change and evolution (de Graaf, 1999).

In summary, the position of development as described in the first section of this chapter, namely as a self-governed process of spontaneous increase in complexity and structure, seems fatally weak. There seems to be no reasonable scientific foundation for such a notion. To the contrary, well-established scientific findings lend support to the conclusion that development should be banned to the realm of romantic philosophical illusions. In the next section, however, we shall see that this conclusion is premature and that the scientific basis of development is stronger than thus far suggested.

In Support of Development: The Dynamic Systems Approach

The Early Years: The Study of Changes Brought About by Interacting Forces

Newton and Leibniz are the fathers of differential calculus, and differential calculus is the mathematical method that allows one to study and formalize continuous change. The fact that motion patterns could be formalized into equations was a major discovery of the seventeenth century. It led to a formalization of the motion of celestial bodies, of pendulums in clocks, of heat transmission in a steam engine. Virtually no domain in which change occurred in some continuous and more or less regular way escaped from study. The common theme was dynamics. *Dynamic* refers to the Greek *dynamikos*, which means 'powerful'. The study of dynamics concerns the way forces apply and how they change and exert an influence on the world. Since Newton, one of the main areas of study was the dynamics of celestial bodies, such as planets or the sun. Planets exert a gravitational influence upon one another – they exchange gravitational force – and by doing so they keep each other in regular orbits, the forms of which were already described by Kepler. Although the dynamics of two interacting bodies, planets for instance, could be formalized and solved without too much effort (speaking in hindsight, that is), the problem of describing the dynamics of three interacting bodies proved notoriously difficult to solve. Two interacting planets form a simple *system* (a word that stems from a Greek verb that means 'to combine') and three planets form a system that is apparently just a little more complicated. However, the truth is that from an explanatory point of view, three planets form a system that is incomparably more complex than two. The so-called three-body problem marked the start of the development of non-linear dynamics as a mathematical discipline. Around the turn of the century, the French mathematician Henri Poincaré developed a set of methods for studying possible solutions of the three-body problem and by doing so laid the foundations of the current science of non-linear dynamics and dynamic systems. Before we proceed, let me point out the possible relations between such vastly differing problems as three planets revolving around each other and the psychological development of human beings. First, both problems concern the mutual relationship between various components that affect one another. Second, both problems concern the evolution of patterns in time – be it spatial patterns or patterns of developing personal properties – that result as a consequence of the interacting forces. The important discovery that Poincaré made was that there exist general methods for approaching those problems, irrespective of their actual content matter. Thanks to these general methods and related insights, dynamic systems theory grew into a general formal approach to the problems of change.

The Study of Stable and Dynamic Equilibria

Further studies in the field of dynamics, both mathematical and physical, demonstrated the existence of spontaneously emerging equilibria. Some systems of interacting forces tend to drive each other to an equilibrium state, that is, a state where the forces involved keep each other at a fixed level or value. This stability is a form of dynamic stability: it is because the forces interact that they keep each other in a locked position. Some forms of stability turned out to be dynamic themselves. For instance, some systems spontaneously evolve towards a cyclical pattern. That is, the pattern of the forces involved keeps changing, but it does so in a cyclical fashion (for instance the so-called van der Pol oscillations that occur in electric and magnetic media). It is even possible for some systems to run into patterns that never repeat but that nevertheless show a high level of regularity. This phenomenon was discovered in the 1960s by a meteorologist by the name of Lorenz who simulated weather phenomena on a computer (although the first experimental evidence for this phenomenon came from the Dutch engineers van der Pol and van der Mark in 1927).

Since the Lorenz model features so prominently in many introductions to chaos and dynamic systems, it is worthwhile to give a little more background information that will also be illustrative of how dynamic systems models operate (see Jackson, 1991b; de Swart, 1990). The weather can be seen as a process of atmospheric circulations (of air with a certain temperature and moisture, for instance) and these circulations can be mathematically represented as the sum of waves with a particular wavelength and amplitude (the fact that a complex wave can be represented as a sum of harmonic waves was discovered by the French mathematician Fourier, 1768-1830, and is used in spectral analysis, which is the basis of the Lorenz and comparable models). In order to provide a reasonably realistic description of the real atmospheric circulation, one needs a model with many such waves. Lorenz wanted to understand the essentials of the interaction between the functions that govern the evolution of such waves and managed to come up with a set of three connected functions. The first describes the magnitude of an atmospheric flow and the second and third describe the magnitude of two temperature waves. It is important to note that this is no longer a model of a real weather system but a model that reduces a weather system (and many comparable systems of flow, such as magnetic flows) to its bare essentials and by doing so tries to understand the fundamental properties of the dynamics. This kind of reduction to the essentials is typical of dynamic systems models, as we will see with the predator-prey model of Lotka and Volterra. The system of equations that Lorenz studied is as follows:

$$\Delta x/\Delta t = \sigma(y - x)$$
$$\Delta y/\Delta t = -xz + rx - y$$
$$\Delta z/\Delta t = xy - bz$$

The parameters σ, r and b are typical of models for dynamic flows; r, for instance, is the so-called Rayleigh number, which is a measure for a temperature difference, for instance the difference between the ground temperature and the temperature at a high altitude.

What is the essential fact or facts about the weather that Lorenz wanted to study with the aid of his three simple equations (and they are indeed very simple, since understanding them requires no more than elementary school mathematics)? A short overview of Lorenz' findings with his three simple equations will show what those essential facts are. First, Lorenz discovered that even simple systems (no more complicated than three mutually interacting variables) could spontaneously settle into regular but never identical patterns, so-called strange attractors (Figure 28.1). Second, he found that such patterns display sudden switches for no apparent reason other than their internal dynamic drive. Third, he found that some interaction patterns – depending on the value of the parameters – are highly sensitive to initial conditions. These are processes we now call by the name 'chaos', although the word chaos itself is quite misleading, since most of what we refer to as 'chaos' are in fact deterministic processes with high apparent irregularity but nevertheless high internal order). If we repeat such a process with only

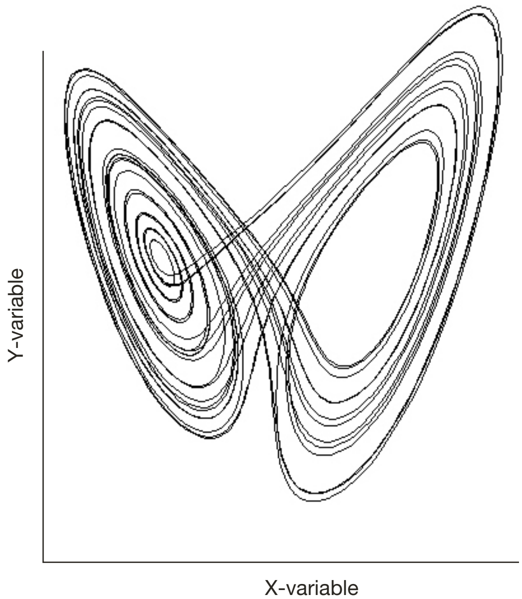

Figure 28.1 *Plotting the position of the* x-*variable against the* y-*variable (or any other combination of the three variables* x, y, *and* z, *for that matter) yields a complicated spatial pattern. The pattern represents the change of the* x- *and* y-*values over time and is known as a 'strange attractor'*

an extremely small difference in its starting conditions, the process trajectory will at first be indistinguishable from that of the first time. But after some time the effect of the minor difference – however minor it may be – will show up in the form of a sudden major divergence between the two dynamic patterns. This is the so-called butterfly effect, which has become almost proverbial: it can be mathematically proven that a butterfly flapping its wings in Singapore, to name just one place, could cause a hurricane in Texas (a finding that puts a heavy burden on the fragile shoulders of innocent butterflies). To put it differently, there exist – simple – systems of interacting forces whose stability is only temporal. After a while they will suddenly diverge and the magnitude of their divergence is in no way proportional to the cause of the divergence. Note that this finding does not imply that all systems of interacting forces are unstable in this particular sense. The point is that some systems tend to stability and will return to that stable point (or pattern) even if they are disturbed – perturbed is the common term – by influences from outside, whereas other systems will vastly diverge if any disturbance occurs, however tiny it may be.

The general lesson that can be learned from these early developments in dynamic systems theory is that systems of interacting forces (can) have a natural tendency to evolve towards some equilibrium state, which is dynamically maintained as soon as it is reached. Some equilibria consist of dynamic patterns, for instance cycles. There exist stable equilibria that are relatively insensitive to perturbations, and unstable equilibria, whose outcomes depend on minute differences in starting conditions or perturbations along the way.

Mathematical Models of Quantitative Biological Processes

Meanwhile, in the 1920s and 1930s, non-linear dynamic modeling was applied to the life sciences, more particularly to biology. The first domains to which dynamic systems ideas were applied were epidemiology – the discipline that deals with the spreading, waxing and waning of diseases – and population biology. Both fields deal with ecological problems, that is, problems relating to the interaction of many different biological species that share an environment. The word 'ecology' is based on the Greek word *oikos*, which means household. An ecological system is characterized by a specific energy flow, by temporal static or dynamic *stabilities* and by long-term change (evolution). Probably the best known example of early dynamic systems modeling is the work of the American ecologist Lotka and the Italian mathematician Volterra. Independently of one another, both scholars discovered the principle of dynamic predator-prey interaction. Volterra's work was based on observations by his future son-in-law, who was a marine biologist. The latter sought an explanation for the finding that during the First World War, when fishing had almost ceased, there was an increase in certain predaceous fish, namely those that lived off prey fish that used to be fished before the war had started. This was an unexpected fact, since one would expect a rise in the population of those prey fish, simply because they were no longer caught in great numbers by fishermen (one could say that the predator fish had taken over the role of the human predators). This question resulted in a simple mathematical model that described the interaction between a predator population and a prey population. Since it is such a nice example of fundamental dynamical thinking, it is worthwhile to go into it a little deeper. Let P be the symbol for the population size of the predator fish (mackerel, for instance) and F the population size of the prey fish (sardines, for instance). If the predators are left without prey, their population will decrease proportionally to a certain death rate, i.e.

$$\Delta P/\Delta t = -aP$$

which one should read as follows: the change in population ΔP over some time interval Δt is equal to the size of the population P multiplied by a rate of dying $-a$ (we could also have set a to a negative number, but this notation is a little more insightful). However, if there is prey to feed on, the predator fish will be able to increase their numbers: the living fish will live longer and newborn predator fish will have a better chance to survive. It is obvious that the increase in the predator population depends on the available prey. If there are a lot of prey fish, considerably more predator fish will survive. Thus, we know that the predator population P increases proportionally to the amount of available prey F and a certain growth rate b, which depends on the predators' natural longevity, their reproductive rate and potentially many other factors that need not be of concern in detail:

$$\Delta P/\Delta t = + bFP$$

In summary, we know that the predators' population is based on two mechanisms, death and survival (including births) and thus we should combine the two equations into one:

$$\Delta P/\Delta t = - aP + bFP = P(-a + bF)$$

However, the predators are not alone in this world: every prey fish they eat affects the food resource on which they thrive. Therefore we must also specify a model of the prey fish population F. We begin with the assumption that the prey fish live off some food source that is independent of the predators and that, thanks to this food source, the prey fish population F increases by a certain survival (maintenance and birth) factor c:

$$\Delta F/\Delta t = + cF$$

Unfortunately, the prey fish are hunted by the predators. The more predators there are around, the more prey fish are caught. Given there are so many predator fish, we can describe the rate of prey catching by a constant d which depends on the predators' hunting skills. It is clear that the more prey fish are around, the more prey fish will be caught, and thus the prey fish population decreases proportionally to the catch rate and the number of predator fish that hunt them:

$$\Delta F/\Delta t = - dPF$$

Similar to the predator fish population, the prey fish population is based on the combination of death and survival, that is, the combination of the decrease and increase factor:

$$\Delta F/\Delta t = + cF - dPF = F(c - dP)$$

We know that the two populations are coupled. Each time a prey fish is caught, the prey fish population changes but so does the predator population, since the eating of the prey fish will increase the chances of the predator to survive. Thus, we have to combine the two equations in a system of equations, where one equation refers to the other and vice versa:

$$\Delta P/\Delta t = - aP + bFP = P(-a + bF)$$
$$\Delta F/\Delta t = + cF - dPF = F(c - dP)$$

This system of coupled equations is a prime example of a dynamic system. It specifies the change in two variables as a function of time, of the preceding state of each variable and of the preceding state of the variable to which it is coupled. This mathematical model results in a series of population sizes over time that show an interesting pattern, which confirmed the Italian observations, namely a series of lagged cyclical changes in the population sizes of both prey and predators (Figure 28.2). The cycles are not caused by some external factor but are entirely based on the dynamic interaction (Hofbauer & Sigmund, 1988; Murray, 1989). This simple model captures an essential element of predator-prey dynamics, namely the cyclical oscillation of populations. It should be noted that it is not meant as an empirical model of actual predator-prey interactions. Rather, it tries to capture the essence of those dynamics by using the smallest and most elementary set of assumptions. A more realistic model can use the Lotka-Volterra model as its starting point and add all necessary assumptions about the populations studied in order to arrive at a model that better fits a chosen part of reality. However, the model illustrates a fundamental feature of dynamic modeling, namely the search for the simplest possible model that is as close as possible to the essence of some dynamic phenomenon.

Another interesting approach concerned the study of the diffusion of certain genes through a population and the potential evolutionary effects of that spreading. An ecological system is a prime example of a self-sustaining, stable and yet developing or changing structure. In spite of its complexity, it is governed by a small number of basic dynamic principles that explain its order and evolution. The dynamic approach to ecological systems may form a source of inspiration for a comparable approach to behavioral and psychological development, as we shall see later.

Computational Approaches to Dynamics and the Emergence of Systems Thinking

During and after the Second World War, an important technological and theoretical breakthrough took place with the development of the digital computer. The study of dynamic systems would be virtually impossible without computers. Many dynamic systems

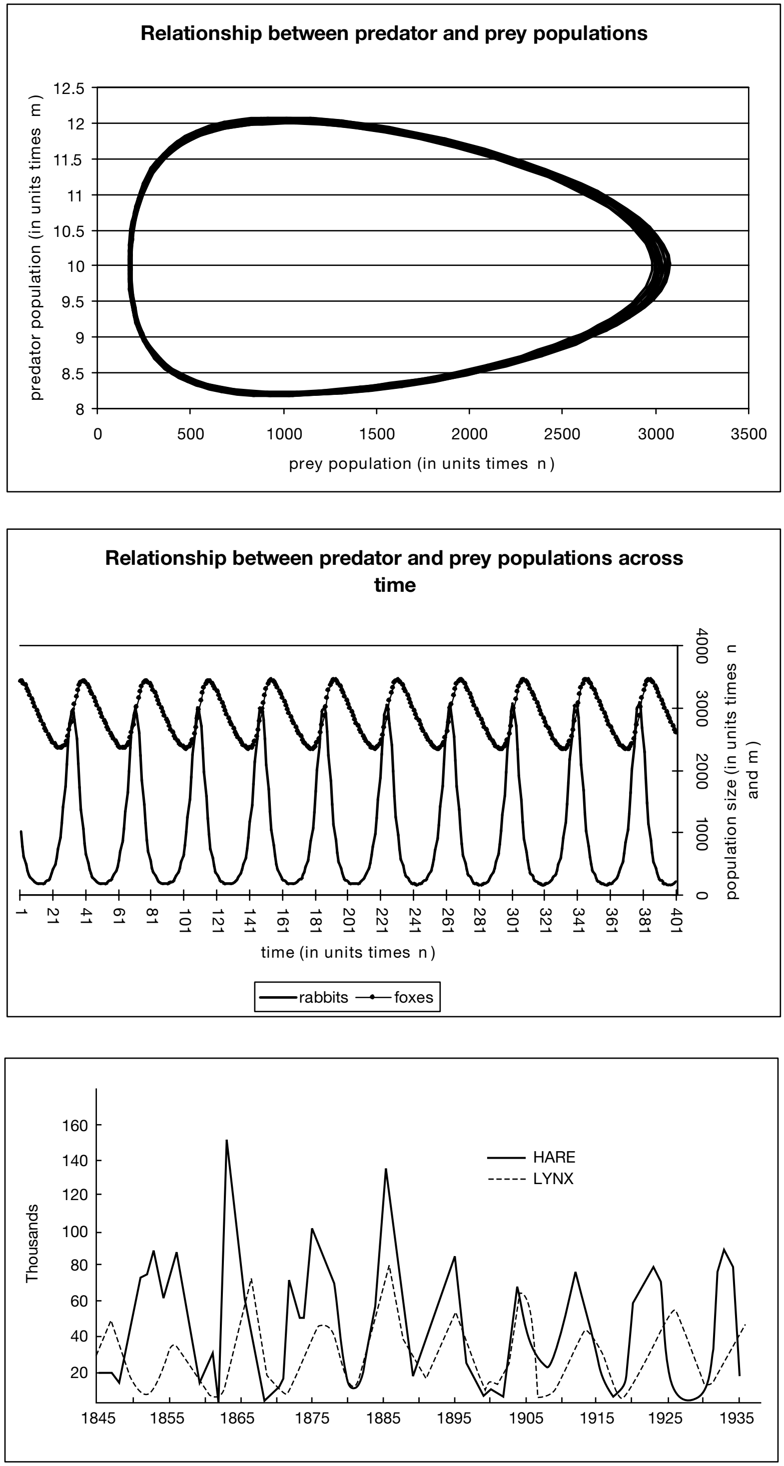

Figure 28.2 *The population dynamics of predator–prey interactions. Relationship (top) between the size of the predator population and the size of the prey population. The egg shape represents the lagged occurrence of (middle) two oscillating patterns. Data (bottom) based on furs of snowshoe hare and lynx (a typical prey–predator species couple) sold to the Canadian Hudson Bay Company in the nineteenth century (Lotka, 1925; Volterra, 1926)*

models cannot be solved analytically. In order to study them, their behavior must be numerically simulated and in order to do so, a computer is an indispensable instrument. Oddly enough, computers are strictly linear and sequential machines and are therefore very different from the dynamic systems that are modeled with them. The coming of the computer also boosted a lot of fundamental ideas about systems in general. In the 1940s and 1950s, researchers like Wiener, von Neuman, Ashby and von Foerster explored topics such as complexity, self-organization, connectionist systems and adaptation. One of their main ideas was that all systems – irrespective of whether they are of a physical, a biological, a social or a psychological nature – display certain general characteristics that capture the fundamental quality of such systems. This idea gave rise to new systems approaches, like cybernetics (Wiener) and general systems theory (von Bertalanffy, Boulding). In psychology, the idea of systems was most notably explored by Herbert Simon (1969), whose work concentrated on hierarchically organized systems that are capable of adaptive information processing. In these earlier systems approaches, the predicate 'dynamic' did not feature as explicitly as it does today. However, from the start, it was clearly acknowledged that systems are, by their very nature, time-dependent and dynamic.

The early and fundamental work on complex dynamic systems diverged into a wide range of topics and approaches. In the 1960s, the French mathematician René Thom (1972) began to study the general properties of sudden changes, more particularly of discontinuities. A good example of a discontinuity from developmental psychology is Piaget's notion of transitions from one stage to another. Thom described a small set of general, elementary discontinuities that he called 'catastrophes' (hence 'catastrophe theory'). The Piagetian-type stage transition, for instance, amounts to a so-called cusp catastrophe, that is, a discontinuous change based on gradual changes in only two control dimensions (van der Maas & Molenaar, 1992) (Figure 28.3).

Another major interest of dynamic systems students is chaos, a theme that became popular with James Gleick's best-selling book (Gleick, 1987). Chaos is a somewhat misleading term, since it refers to patterns that are, on the surface, extremely disorderly and random, but that in reality show a deep underlying order. The most important feature of chaos is probably that it can emerge spontaneously as certain variables that control the behavior of simple, orderly systems cross a specific threshold value. For instance, when the reproduction rate of biological populations that have discrete breeding seasons, insects for instance, exceeds a certain value, the population sizes start to oscillate in a seemingly random, chaotic way (May, 1976). Typically, chaotic systems are highly sensitive to initial state conditions and exhibit the butterfly effect discussed earlier. The discussion about the eventual importance of chaos to developmental psychology has not been settled yet. In order to empirically demonstrate that a process is really chaotic and not just driven by a multitude of independent external factors, one needs quantities of data that are usually beyond the reach of developmental research. However, chaos theory has shown that randomness and chaotic variation do not need to come from outside the system. They can be produced by the system itself if the conditions are right. In development, variability can be an important functional aspect and it is important that such variability can be produced by the developing system itself, i.e. that it is not necessarily dependent on external factors (de Weerth, van Geert, & Hoijtink, 1999).

Epigenesis and the Emergence of Biological Form

In biology, the 1920s and 1930s witnessed the birth of mathematical biophysics, which unraveled a number of interesting dynamic principles. During the 1950s and 1960s, important ideas about the dynamics of development were initiated by a number of biologists interested in developmental biology and embryogenesis (see Gottlieb, 1992, for an overview and discussion with applications to developmental psychology; and see Gottlieb, Chapter 1 in this volume). Probably the best-known representative of this approach is the British biologist and embryologist Conrad Hal Waddington (1905-1975), whose picture of the epigenetic landscape – showing a ball rolling down a landscape of hills and valleys – features in almost every textbook on developmental psychology (Figure 28.4).

Waddington's basic contribution to the dynamic thinking about development can be contrasted to the widespread but simplistic view that genes carry the full description of the organism's form, or more precisely that the genes contain a full set of instructions for how to build a body up to its finest details (for instance, the Habsburg kings all had the same remarkable, somewhat protruded chin; it can be assumed therefore that the building instructions for that chin must be contained in the Habsburg genes: where else could that chin come from?). Waddington showed that genes form the starting point of embryogenesis and that the process of embryogenesis itself creates the conditions under which the organism's body plan comes about. Simply said, genes may code for the production of certain tissue, but once that tissue is formed, it may cause other tissues to develop, or it may cause certain genes to turn on or off. This basic idea, that the form of the body is literally constructed by the construction process itself – and is not specified in some pre-existing full instruction set, design or

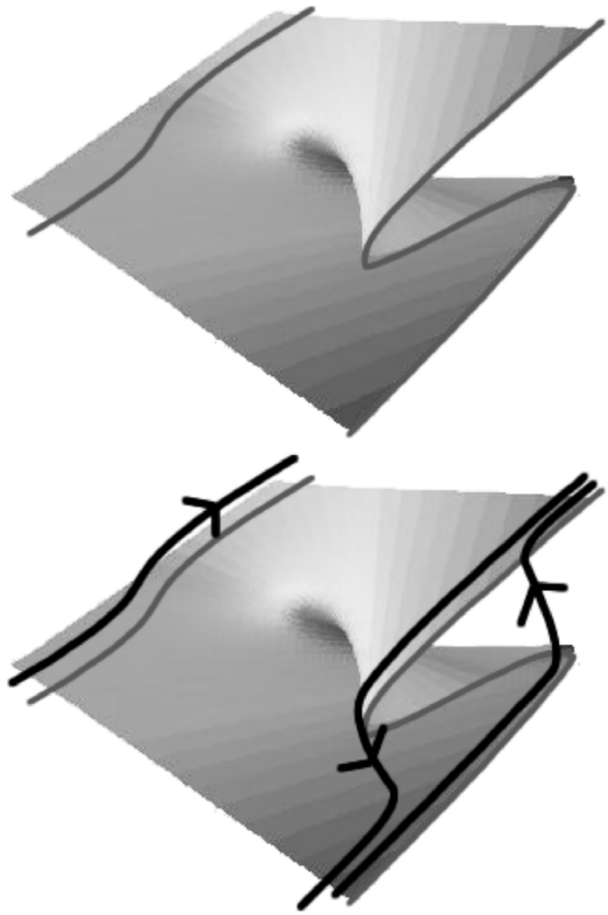

Figure 28.3 *The cusp catastrophe refers to a particular kind of discontinuous change. The folded surface represents different transition possibilities (grey lines). The transition is either continuous (top) or discontinuous (bottom). Which type of transition will occur depends on the values of the control parameters that govern the process at issue. The cusp catastrophe is a general model of processes based on two control parameters*

building plan – is known under the term *epigenesis*. The epigenetic concept from embryology currently features as an important dynamic metaphor for students in the field of developmental psychology (de Graaff, 1999; Gottlieb, Chapter 1 in this volume). A simple example may help to clarify the gist of this approach. Suppose you arrive at the station of some big, unknown city and you have to go to a particular place. You can use a city map to get there. The city map gives a complete description of all the streets you have to go along and so functions as an explicit, predefined instruction set. Suppose you didn't have a street map and you had to ask a passer-by. He or she could sum up all the turns you had to take and all the street names you had to remember in order to get to your destination. Again, the description is a full instruction set but it has one major disadvantage: it is difficult to remember. You'll probably forget the order of the lefts and the rights and you'll get lost. One thing you could do is remember the first half of the instructions, act upon them and when you've reached the last instruction ask another passer-by for the additional instructions. This situation is comparable to dividing the developmental instructions between a set of genetic and a set of environmental instructions. Together they fully define the developmental path. But suppose the city has a number of big squares connected by major streets or avenues. In that case, your informant might say, 'Go straight ahead to Square X, and there take the major avenue to your right and follow that to the next square, and there you do the same thing and you'll automatically get at your destination.' In this particular case, the meaning of the information given

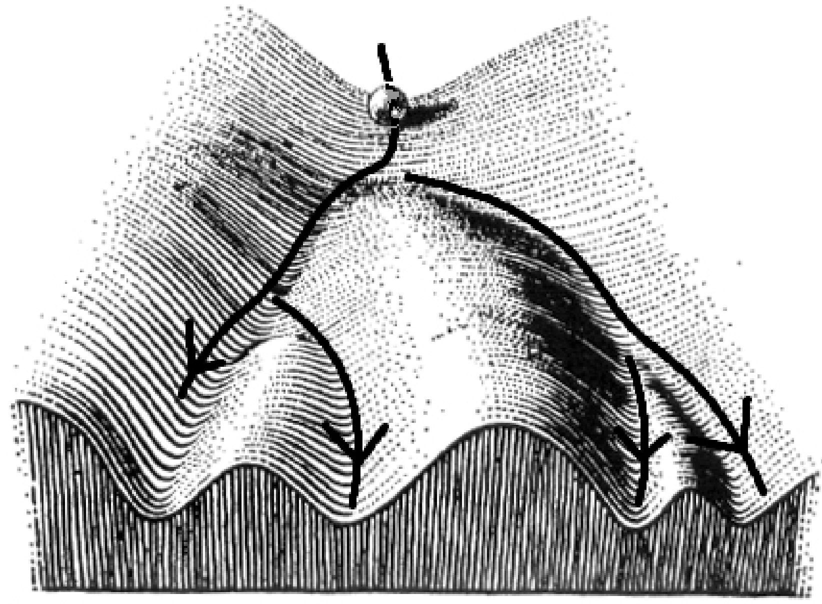

Figure 28.4 *Waddington's epigenetic landscape. The position of the ball represents an organism in a space of developmental (e.g. morphogenetical) dimensions or possibilities.The hills and valleys represent different developmental pathways. Downhill pathways require less energy and will be more common than (slightly) uphill pathways. Steep uphill pathways are in principle non-existent (their probability is too small) (after Waddington, 1956, p. 412)*

depends entirely on what you'll find once you arrive at each of the major squares. You'll have to get there first in order to know what choice of road you'll have to make. The information becomes available as you carry out the simple instruction given at the beginning. This latter situation is somewhat comparable to the epigenetic explanation of development. Each step in the process creates the conditions for the next step. In fact, there exists a kind of *bidirectionality* between the traveler and the city. The traveler follows an instruction, which brings him to some particular place, and once arrived at this place it becomes clear what the next instruction should be.

Note that the traveler can make mistakes, for instance when two avenues he'll have to choose from are about as broad and both to the right. But since he knows the general principle, he can always retrace his steps or make a detour if need be. In other words, the itinerary is defined in a *probabilistic* way. At every point, the traveler has to decide which step is the most probable, given the local circumstances. It is highly likely that another traveler, following the same general instruction, will also reach the same destination but through a somewhat different itinerary. Biologists working in the epigenetic tradition call this *equifinality* – reaching a similar goal through different means or paths.

Note also that the success of the epigenetic solution to finding your way in the city depends very much on the structure of that city, and some cities may be easier to walk through than others. Thus, if someone tries to understand my itinerary, he or she has to take account not only of the instructions given to me but also of the city plan and how it looks to the pedestrian. That is to say, one has to take a holistic view of the problem. This position was defended by the theoretical biologist von Bertalanffy, who coined the term general systems theory, as a general account of problems of complex order. The holistic view is consistent with the bidirectionality mentioned earlier, which implies that traveler and city in fact interact and by doing so produce a successful route to the final destination.

Spontaneous Increase of Order and Structure

One of the recurrent themes in the dynamic biological systems view is that development is characterized by an increase of complexity and by the creation of novel forms, that is, forms (properties) that were not explicitly specified or coded for in the initial state (Gottlieb, Wahlsten, & Lickliter, 1998). The idea of increasing complexity has been pioneered in non-linear thermodynamics, especially in the field of chemistry. In the 1960s and 1970s the Belgian chemist (of Russian descent) Ilya Prigogine studied chemical reactions that self-organized into complex patterns that maintained themselves as long as a sufficient energy supply was administered. We have already met the second law of thermodynamics, which – highly simplified – says that energy spontaneously streams from hotter to colder objects and never the other way around. The second law implies that if one starts with a world with concentrated spots of heat (hot objects, like a cup of fresh coffee on my desk), the result will inevitably be a world with a diminished concentration of heat (with the coffee having the same temperature as the air in my office), or, in the end, a world with a completely uniform distribution of temperature. If one identifies the specific concentration of heat with a high amount of structure (or specificity), it follows that structure must decline spontaneously (the distribution becomes more even). In a technical sense, we can say that the world is characterized by a spontaneous increase of entropy, which, for our purposes, can be simplified as a spontaneous loss of order or structure (note that entropy has a specific physical definition, but explaining this is beyond the scope of the present chapter: see Atkins, 1984). What holds for temperature also holds for information (in the formal, mathematical sense, both notions are highly similar). A practical application of the second law to the field of information is that if one sends a message to someone else – over a phone line, for instance – there is always a spontaneous loss of information. The line is noisy and some words are difficult to understand. The opposite never occurs spontaneously: there is no telephone line that spontaneously transforms a noisy message into crisp, clearly understandable words. One of the major findings of non-linear thermodynamics, as studied by Prigogine and many others, is that there exist processes that appear to contradict the second law: they do result in an increase of order, structure or information. There exist chemical reactions, such as the Belousov-Zhabotinsky reaction, that spontaneously produce complex spatial and temporal patterns. They are the result of an *autocatalytic* process. In such a process, the reactants produce a chemical compound that facilitates the formation of another compound, that eventually either counteracts or facilitates the first, or affects still another one, and so on. As a result, the process oscillates between complex, spontaneously produced states. The only thing we have to do to keep the process going is to give it a constant supply of some basic reactant or temperature.

The output of the process – the spatial or temporal patterns formed by the chemical reactants – is considerably more complex than the input (Figure 28.5). This is a result of *self-organization*: the process organizes itself into complex patterns. There is nothing that instructs the process to do so: it spontaneously creates itself. Self-organization occurs in processes or systems that already have a high amount of structure by themselves. That is, it occurs in complex systems. But don't these systems violate the second law of thermodynamics? In the end, they do not: the spontaneous creation of structure in such systems actually increases the flow of energy through such systems. They exist by virtue of an increasing loss of structure elsewhere in their environment (they dissipate energy, which is also why they are called dissipative systems).

The existence of spontaneous self-organization and the general conditions under which it exists is an important discovery, demonstrating that increase of order is a natural and basic phenomenon of nature. This fact as such does not prove that psychological and behavioral development is also such a process, i.e. one where order and structure are created, but given the generality of such processes, it would be remarkable if development were not self-organizational. It should also be noted that self-organization and an increase of order and structure occur at the cost of increased energy consumption. Development, if it is indeed a self-organizational system, consumes energy (or information, which is basically the same) and is therefore confined by the available energy (or information) in the environment.

The Dynamics of Complexity

Developments in the 1980s and 1990s shifted the interest from the issue of increasing order and structure *per se* to the question of how and why order and novelty emerge. Complexity became the major theme. A complex system consists of a large amount of elements or components that interact with one another. A physical example of a complex system – although it doesn't sound like one – is a heap of sand to which new sand is added (like the heap of sand made by a child who digs a pit on the beach). The grains of sand exert a certain amount of force on one another because they are all subject to the forces of gravity and friction. As a result, the sand in the heap slips down in the form of avalanches of different sizes. It is interesting to note that the study of heaps of sand brought one investigator – Per Bak – to the discovery of an interesting dynamic phenomenon, namely *subcriticality* (Bak & Chen, 1991). Subcriticality is the state that keeps dynamic systems on the verge of changing and that causes sudden changes or discontinuities at different levels of magnitude (Adami, 1995). We have already encountered the issue of sudden changes in our short discussion of catastrophe theory and how it eventually related to stagewise developmental change.

Complex systems are widely studied in the field of biology. An example of a complex system is an

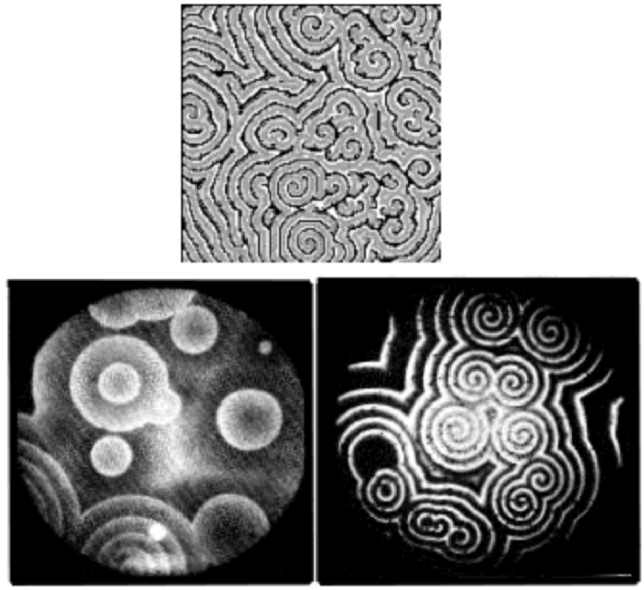

Figure 28.5 *The Belousov–Zhabotinsky reaction, discovered by Belousov in 1958 and described by Zhabotinksy in 1964. The reaction is based on the interaction of four chemicals (sulfuric acid, sodium bromate, malonic acid and ferroin). The chemicals produce a reaction pattern of spatially distributed waves in the form of a chaotic pattern or strange attractor. The reaction is one of the standard examples of self-organization in chemical reactions*

ecosystem: the arrangement of animal and plant species that interact with one another in a particular time and place. Another example is the web of interactions between different biological species. Complexity theory has been used to explain the processes of biological evolution and extinction, particularly by Stuart Kauffman. Kauffman (1993) studied general aspects of evolution in networks of biological species by reducing species to single on-off nodes in a so-called Boolean network. Although this approach amounts to an incredible reduction of the essence of a biological species, it nevertheless captures essential aspects of the evolutionary process. We have already encountered this important aspect of the dynamic systems approach, where a seemingly unacceptable amount of simplification is often the key to understanding the deepest aspects of the processes under scrutiny. An interesting finding in complex systems studies, such as Kauffman's networks of species, is that one gets order and structure for free – that is as a result of spontaneous self-organization – only in complex systems within a certain range of interconnectedness. To see what connectedness means, imagine a social group consisting of many people. If all people are in some way interacting with each other, the degree of interconnection is complete. If no person interacts with someone else, connectedness is minimal (zero in fact). If every person interacts with just a few others, connectedness is low, but often sufficient to get interesting self-organizational processes off the ground (more interesting than those occurring with complete interconnectedness, for instance). Although the degree of connectedness seems at first like a rather trivial property of a system, it is nevertheless critical to the emergence of self-organization. This point illustrates another aspect that is often found in dynamic systems, namely that properties that at first seem trivial and irrelevant to the process at issue can nevertheless play an essential role (which is not to say that all trivial properties are essential, of course, or that essential roles are always played by trivial properties).

From Complexity to Connections

Another widely studied biological example of a complex self-organizing system – that is also closely related to the field of psychology – is the brain. The brain consists of a very large network of interacting neurons, each neuron directly connected with only a limited number of other neurons. The study of such networks has led to the development of an exciting new field, namely that of connectionist networks – also called artificial neural networks – that are used to explain and simulate processes of learning and pattern formation (Bechtel & Abrahamsen, 1991). A connectionist network basically consists of a large number of simple components. The role of each component is to receive input from other components and send some output back (or send it to the outside world). The inputs and outputs are simple: they usually consist of an activation level, which can be represented by a number (e.g. an output or input of magnitude 5 represents a higher activation level than an input or output of 2). The output of a component (the activation it sends to other components or to the outside world) is a function of the input it receives (the activation it receives from other components). The total input a component receives is, in principle, a simple sum of the inputs it receives from all the components to which it is connected by an input channel. Thus, a component that receives a total input of, say, 10, will send out a higher activation level than a component that receives a total input of 3, for instance. A central feature of a connectionist network is that every connection between two components carries a specific weight (think of that connection as a channel that links one component to another and through which the activation flows). The function of the weights is to alter the magnitude of the activation levels sent out by the connected component. For instance, if a component A sends out an activation level of 5 to a component B and the A-B connection channel carries a weight of 2, B receives an input of 10; if the weight is -1, B receives an input of -5 (and this input will be subtracted from the inputs coming from other components). The importance of connectionist networks lies in the fact that they are adaptive, that they can learn. A network may receive inputs from the outside world, for instance visual information about forms of objects. In its turn, it may send an output to that outside world, for instance a name of the object it 'sees'. Before it can do so, however, it has to learn which words should be associated with which visual inputs. If the system improves in making the correct association, it shows learning or adaptation. This is exactly what connectionist networks are made for, namely to adapt to the requirements of some environment, i.e. to learn. They do so by altering the weights between the components. The idea is that if an output is consistent with some criterion (e.g. the name of the object chosen by the network, given a specific visual input, is also the name approved by the environment) the weights between the components that have led to this correct output are amplified (if the output is wrong or inconsistent, the weights are diminished). In general, the weights are altered by a function that is proportional to the distance between the given output and the desired output (the bigger that distance, the bigger the error). The most important feature of connectionist networks – or artificial neural nets – is that the learning or adaptation occurs automatically, that is, as a result of self-organization. We don't have to penetrate the system and change the weights and output functions by hand, so to speak, in order to get the system into the right direction. Connectionist systems will find the rules or the patterns by themselves. Furthermore, the input does not need to be entirely consistent. Even if the environment makes mistakes or if it is to a certain extent inconsistent in its corrections or in its inputs, the connectionist network will nevertheless pick up the required rules, associations or adaptations.

Why are connectionist network models important to the study of development? The answer is that those models actually provide a proof of the claim that learning can occur in networks of simple interacting components that have some sort of connection with a structured, outside world (Elman et al., 1996). Superficially, this may seem a trivial accomplishment, were it not that before connectionist modeling became available, we had no proof for the contention that – among others biological – systems could spontaneously construct order and regularity, given an input that exemplified the structure or order in some way or another. That is, knowledge – of whatever sort – does not have to be present or programmed in advance; it can emerge spontaneously, given the right sort of input. Although learning and adaptation seem almost trivial (why would anyone doubt their possibility), connectionist models provide the first real demonstration of the basic 'mechanical' conditions under which those seemingly trivial accomplishments occur. It needs to be said, however, that the fact that connectionist networks represent a form of learning and adaptation does not logically imply that all forms of learning and adaptation must occur by connectionist network principles. An important feature of those models is that they are 'brain-like' in an abstract sense. Both the brain and connectionist networks consist of interconnected units that receive and produce levels of activation and by doing so can accomplish extremely complex processes of pattern recognition, classification, association, and so forth. They operate under conditions that require a high amount of error tolerance, incompleteness, inconsistency and fuzziness, which is characteristic of systems that need to operate under natural, biologically valid circumstances. At present, connectionist networks are among the most studied examples of complex systems, that is, systems consisting of interconnected simple units that self-organize and by doing so perform complex symbolic tasks.

In Support of Development

In the first section we explored the colloquial meaning of the concept of *development*. We found that it entailed aspects of unwrapping inherent potentialities, guidance by an intrinsic tendency towards higher complexity and the construction or production of novelty. In the second section we discussed scientific approaches according to which this image of development is only an illusion. We saw that increase of complexity is impossible and that the emergence of structure must be based on pre-programmed, full instruction sets, such as genes or transmittable contents. In the third section we turned to developments in the field of dynamic systems theory and found that the conclusions of the second section were, at best, preliminary. In complex systems, increase of structure and complexity seems the rule rather than the exception. Self-organization, a process of creating structure and order without explicit instructions or guidance from outside, is a general mechanism and we have now come to understand some of its basic properties and possibilities. The romantic image sketched in the first section did not seem too far off the truth after all; or, to put it differently, there exist no logical or empirical impediments for it. More importantly, dynamic systems theory has provided a number of conceptual, mathematical and methodological tools by which complex, self-organizing processes can be described, explored and studied. The systems approach emphasizes the fact that important properties of systems can be studied irrespective of the actual, physical or other properties of the systems at issue. Principles that govern physical or biological processes can also be applied to social and psychological processes, provided that similar general properties hold. These properties are usually related to the way the components in the system interact and how they change, that is, to the dynamic aspects of those components. In the next section I shall explain how dynamic systems thinking can be applied to the problems of psychological and behavioral development and how dynamic systems models of such developmental processes may be designed.

A Dynamic Systems Approach to Development

Dynamic systems theory is an approach to perceiving, conceptualizing and studying phenomena and events we find of interest. It consists of a collection of general concepts, methods and techniques and of an ever-increasing series of worked-out examples in a variety of fields. In the preceding section, I have introduced a number of those general aspects and examples in an attempt to provide a first, intuitive grasp of what dynamic systems theory is about. As a collection of generally and widely applicable tools, dynamic systems theory can also be fruitfully applied to developmental psychology. This assertion does not imply that its application will lead to results that are comparable to those from fields such as physics, chemistry or biology. The latter are sometimes light years ahead of developmental psychology in terms of data collection and mathematical rigor, but that does not prevent us from making dynamic systems approaches work for developmental psychology too.

What Is a Dynamic System of Development?

The Universe of Discourse

In accordance with the basic principles of dynamic systems theory itself, the best way of explaining the nature of a dynamic system is to show how to get at one. In order to construct a dynamic systems model of development, one starts with selecting a universe of discourse. This universe of discourse is a vaguely confined, highly implicit collection of phenomena, concepts, approaches and so forth that relate to the content matter of developmental psychology. It is basically what the community of developmental psychologists as a whole understands by 'development'. No single person has a copy of this universe of discourse in his or her head, but as a community we understand each other more or less and know more or less what we are talking about. Note that this universe of discourse is considerably vaster than the topics and concepts that developmental psychologists address at this very moment. Some regions, so to speak, are more frequently visited than others (there's more interest in the development of the object concept, for instance, than in the development of dreaming: Breeuwsma, 1993). Some people prefer to stay in one region of the universe of discourse, others prefer entirely different spaces (cognitivists, for instance, consider concepts and representations real entities of development, whereas ecological psychologists and dynamic systems theorists inspired by ecological psychology stick to action and perception in real time and consider concepts and representations a myth). The universe of discourse changes as new insights and viewpoints wax and wane over the years. Currently, for instance, developmental discourse relies heavily on biological, genetic and neurological concepts and phenomena; in the early 1970s it was strongly influenced by the narrative of cultural and social influences. A deliberate choice for a specific universe of discourse is hardly ever made. It is implicit in our research, discussions and theorizing. Nevertheless, it forms a crucial backdrop against which all our efforts at understanding development make sense.

The Dynamic System

A system is basically any collection of phenomena, components, variables or whatever that we take from our universe of discourse that we are interested in. This collection is a system in as much as its components relate to one another. It is a dynamic system if its components affect and change one another in the course of time. In order to make clear what this means I shall start with an example that is not a dynamic system but that is nevertheless recognizable as an example of common theory formation in psychology. Let us assume we are interested in the relationship between social economic level (SES) of families and the intelligence (IQ) of their children. The SES-IQ collection is a system because it consists of components that relate to one another. For instance, it is claimed that higher SES corresponds with higher IQ, on average. Is the SES-IQ system a dynamic system? It is not, in spite of the fact that we can say that, somewhat loosely stated, SES affects IQ. What we mean by that claim, under the standard interpretation, however, is that in a particular population SES relates to IQ in such a way that IQ can be statistically predicted, given SES. It is a static system in that it specifies that for any SES level there exists a certain, loosely confined cloud of IQ levels (and it is this cloudy character that will be expressed in the mathematical format of a correlation, which we are all used to).

How would a dynamic systems theorist look at the SES-IQ model? In order for a system to be dynamic – that is a system where the components affect one another over time – we have to specify a format of actual dynamic influence. SES is a bookkeeping term, basically. It is not something that actually features in real interactions between people. However, we know that SES corresponds with things like schooling, knowledge, interaction style and so forth and that these things correspond with the language used, the quality of intellectual help given to children and so forth. IQ too is basically a bookkeeping term. Yet we know that it relates to actual things such as problem solving, knowing and understanding a wide variety of words and so forth. Let us take a factor that is related – maybe somewhat loosely – to the sociological concept of SES, namely the verbal interaction patterns between parents in children. We now take another factor that is also related to what we usually call IQ, namely language understanding and the ability to reason with verbally presented concepts. It is reasonable to assume that, all other factors being equal, elaborate verbal interactions between parents and children will enhance the child's verbal understanding and verbal skills. Thus, for some arbitrary moment in time we claim that

$$\Delta V_c/\Delta t = aV_cP_p$$

by which we mean that the increase in the verbal skill of a child V_c is a function of the verbal skill the child has already acquired, the verbal performance P_p of the child's parents and some parameter a that regulates the effect of parental verbal performance on the child's verbal skill. If we assume that the parent's verbal performance is a direct product of the parent's verbal skill, we may simplify the equation as follows:

$$\Delta V_c/\Delta t = aV_cV_p$$

It is also reasonable to assume that, all other things being equal, the child's verbal skill will have some effect on the parent's verbal performance, and thus on the parent's verbal skill in the broad sense of the word. Thus, if the child easily understands what the parent says and asks for more justifications of the parent's disciplinary measures, the parent will, in many cases, be obliged to adopt a more elaborate style of verbal justification. We can specify this relationship in the following equation:

$$\Delta V_p/\Delta t = bV_cV_p$$

Substituting x for V_c and y for V_p we can write our model as a set of connected equations:

$$\Delta x/\Delta t = axy$$

$$\Delta y/\Delta t = bxy$$

which is a dynamic model that is comparable to those we found in biology (the predator-prey equations) or physics (the Lorenz equation for flows in atmospheric systems). Of course we do not know whether our toy model of mutual interaction between parents and children has anything to do with reality, but that is a different matter. The point is that we have constructed a dynamic systems model consisting of two components, namely the verbal skill or habit of a parent and that of a child. The model says that, first, for every temporary level of verbal skill or habit in a parent and a child, there is some effect on a succeeding state of verbal skill or habit in that parent and child; and, second, that this effect is moderated by two parameters, a and b.

More often than not, this collection of components we are interested in is relatively vaguely specified. For instance, people interested in the development of the early object concept call upon a somewhat loosely specified set of concepts and phenomena, such as object concept, object permanence, hiding and finding objects, reaching and grasping, that are all supposed to relate to the notion 'early object concept'. New research in the field usually results in extending or confining the collection. For instance, when it is found that hiding time and number of objects play a role in the infant's successful retrieval of a hidden object (Wellman, Cross, & Bartsch, 1987), memory and perception become aspects that relate to the object concept in a relevant way.

Once the results of empirical research have to be communicated, however, the loose collection of components is reduced to a strictly confined set of variables that relate to one another in some statistically specified way (for instance, the length of the delay between hiding the object and letting the infant search is correlated to the infants' retrieval success, as specified across a sample of actually studied infants). A similar reduction and specification of the components also results from an attempt to turn the original conceptual model (of the object concept, for instance) into a dynamic systems model that is used for simulation or numerical experimentation.

The Environment

Once we have specified a dynamic system within our universe of discourse we have also implicitly specified another component, namely the system's environment. The system's environment is everything in the universe of discourse that does not belong to the system but nevertheless interacts with it. In our historical overview we have discussed thermodynamic systems that spontaneously increased their internal structure by consuming energy from their ambient environment. By doing so they reduced the amount of order in the wider, ambient environment and so complied with the general entropic principle that governs the whole of nature. It is important to note that both environment and system result from the choices made by the researcher: they are not implicit categories of nature. Take for instance the following specification of a dynamic system. One component of the system is a caretaking adult who is also a mature and competent speaker of a language. The other component is an infant who is not yet in command of that language and will learn it from the caretaker. This two-component system defines an environment, which is basically everything else that affects both the infant and the adult. Note that we shall confine ourselves to only those environmental influences that are related to the dynamic process at hand, which in this case is one of language transmission and appropriation. It goes without saying that the real interactions between the wider environment and the adult-infant system are of incredible complexity. However, we do not need to accommodate all this complexity in order to understand how the system and the environment operate. Systems thinking is basically about finding the right simplifications, that is, those simplifications of reality that are necessary to capture the basic aspects of the dynamics at issue. We have given a biological example – that of a predator-prey relationship – in which the relationship with the broader environment could be reduced to a constant inflow of energy. This energy is whatever the prey population needs to sustain itself. This simplification suffices to specify the most basic and important properties of a predator-prey system.

The same principle holds for the adult-infant system and the environment as defined by that. It suffices that we treat the environment as a source of otherwise unspecified energy needed to let the adult-infant system 'run'. A more appropriate term than energy is the term *resources*. Thus we see the environment as the origin of all the resources required to let the dynamics of the adult-infant system unfold itself. The only thing that we should really reckon with is that the resources are limited or constrained (van Geert, 1991). For instance, in order to transmit and learn language, the adult-infant system requires time, working memory, a language to transmit or learn, attention and effort and so forth. These resource components may be big, but they are also limited. The participants' working memories, for instance, are extremely limited in comparison to the complexity of the content – the language – that is produced in the adult-infant dynamics. The role of limited general resources is not always (in fact, mostly not) accounted for in psychological theorizing and research. It is, however, crucial from a dynamic point of view (Elman, 1994; van Geert, 1991). Since a further discussion of this issue far exceeds the scope of the present chapter, suffice it to say that the intrinsic resource limitation is an important driving force of any dynamics, explaining, among other things, the emergence of equilibria (I shall come back to this later when I present an example of dynamic model building based on principles of resource-dependent growth).

Note that this technical, systems notion of environment differs from what (developmental) psychologists usually call 'environment'. By 'environment' is usually meant the person's objective surroundings: the physical space in which the person lives. In addition, the environment is also often seen as an independent source of influences on the subject, that is, as a collection of forces that can, in principle at least, be freely manipulated. In dynamic systems models, however, the notion of environment is a technical concept, defined, as I explained earlier, by how the system at issue is defined. For instance, if we specify a system consisting of an interaction between two components, namely an immature grammar (for instance that of a 2-year-old language learner) and a mature grammar (for instance the grammar of the language spoken in the community of competent language users), we define the environment as everything that interacts with this two-component system. For instance, the working memory of the language-learning child, which interacts with the language-learning process, is therefore part of the system's environment (in spite of the fact that it is 'inside' the language-learning child). If we split the formerly defined system up in its constituent parts and focus on the former subsystem consisting of the immature grammar, the other former subsystem (the mature grammar) automatically becomes part of the first system's environment. However, since we expect that both former subsystems (the immature and the mature grammar) will actively interact (for instance, the mature grammar will produce a language type called 'Motherese' that is adapted to the learning needs of the immature grammar) it is probably wise to make a distinction between those parts of the environment with which the system actually interacts and those that are basically passive (or that can at least be treated as such for the sake of model building, like working memory, for instance). That is, there is a distinction, in terms of the models employed, between the active environment and the background environment. By making this distinction, we refer to the fact that systems are hierarchically organized (Bronfenbrenner, 1979). The fact that considerable parts of the environment are in close interaction with the developing system implies that the effective environment co-depends, so to speak, on the developmental process. That is, it is not the independent, freely manipulable source of possibilities that it is often seen to be in the standard view. The tendency to view the environment as an objective, independent entity has led to the misrepresentation of family environments, among others. In behavior genetics, for instance, a distinction is made between siblings' shared and non-shared environments, which are nevertheless both part of the same family environment (Pike & Plomin, 1996).

Development in a Dynamic Systems Frame

How Does a Dynamic System Work

A dynamic system – however it is defined – changes because it is affected by other systems (in short, the system's environment) and by itself. The latter aspect is of crucial importance. Let us take as an example of a system a language-learning child. In order to conceive of the child as a dynamic system, we have to follow its changes on a moment-by-moment basis (which is a conceptual choice; it goes without saying that we cannot do so empirically, for instance in the form of a continuous observation of the language acquisition process, which would be just too demanding on the child and the researcher.) At any particular moment, the system is affected by whatever environmental inflow occurs at this particular time, and, equally importantly, by the system's preceding state. This property turns the changes that the system undergoes into what is called an *iterative process*. An iterative process takes the output of its preceding state (that is, the change it underwent in the immediately preceding moment) as the input of its next state. Although it is hard to conceive of a system that is not affected by its preceding state, this iterative property is hardly ever taken seriously in, let us say, standard approaches to development. It is likely that it is considered so trivial that almost no one ever expected anything interesting from it. However, dynamic systems modeling and research has shown that it is exactly this iterative property that explains a lot of the interesting features of changing and developing systems.

For one thing, dynamic systems are (often) non-linear. Mathematically, a linear operator L is defined by the property of linear superposition. This basically means that L is a linear operator if

$$L(ax + by) = aL(x) + bL(y)$$

for a and b constants and x and y functions. A function is a way of associating 'objects' in a set to other 'objects'. For instance, multiplication by 2 is a function that associates a number (an object in the set of numbers) to another object in that set (another number, which is twice the first number). An operator is a symbol that instructs you to do something with what follows the operator. Thus, if 'times 2' is an operator and 'raise to the second power' is an operator, the first is a linear operator and the second is not, since

'times 2'(2 + 3) = 'times 2'(2) + 'times 2'(3)

whereas

'second power'(2 + 3) <> 'second power'(2) + 'second power' (3)

(see Jackson, 1991a, for formal definitions). In short, (non-)linearity is an abstract mathematical property and it should not be identified with the contrast between relationships that can be represented by a straight line and those that can be represented by a curved line, for instance. Nevertheless, it is possible to obtain an intuitive understanding of the non-linearity of dynamic processes. Non-linearity means, among other things, that the effect of a dynamic process differs from the sum of its parts (it can be more but it can also be less, dependent on where in the process the effect is occurring). An alternative and somewhat more intuitive way of defining the property of non-linearity is to say that the effect of a factor that influences the system is not (necessarily) proportional to the magnitude of that factor. For instance, research on the effect of birth weight on later intellectual development has shown that birth weight (e.g. as a consequence of prematurity) has hardly any effect on later development if the infant's weight is above some threshold weight (Wolke & Meyer, 1999). Once it is lower than the threshold, a strong negative effect occurs. This threshold effect is related to the fact that dynamic systems evolve towards some form of (dynamic and often temporal) equilibrium. This means that such systems are 'attracted' towards some end state. The state to which they are attracted, that is, towards which they spontaneously evolve as a consequence of the underlying dynamic principles that govern their behavior, is called the system's *attractor*. Research on dynamic systems in general has demonstrated that attractors can take various forms. The simplest attractor is the point attractor, which implies that the system evolves towards a stable state (like a thermostat that keeps the room's temperature constant). An example of such an attractor is the adult speaker's stable level of linguistic skill. Still another example is the overall developmental state of a person (for instance the concrete operational state that 6- to 12-year-old children are supposed to occupy according to the Piagetian model). Another type of attractor is the cyclical attractor, which implies that the states of the system are running through a cycle. An example of such an attractor can be found in the neo-Piagetian stage theories, which assume that every stage is characterized by a repetitive cycle of substages (Case, 1990). We should realize, however, that the attractors of complex, natural systems are far less regular than those found in mathematically pure systems and that the latter are, at best maybe, only metaphors of the complex equilibria of natural systems.

Dynamic systems are affected by control variables. An example of a control variable in a population of animals is the animals' average reproduction rate or their average longevity. An example of a control variable in a cognitive system is the size of the system's working memory. Limits on working memory size may affect the final stage of cognitive development that the system may reach. Scaling up a control variable – a gradual increase in working memory due to neurological maturation, for instance – may result in the system making an abrupt choice between either of two mutually exclusive states. For instance, children confronted with a Piagetian conservation experiment either understand the conservation principle (state B) or not (state A). It has been hypothesized that a gradual increase in working memory, for instance, will result in a relatively abrupt appearance of conservation understanding. More precisely, a system that had only one possible state (non-conservation A) has now two (non-conservation A, and conservation B). The state the system will actually occupy (A or B) will depend on, for instance, the nature of the conservation problem they are presented with (van der Maas, 1993). Points – or better, conditions – under which such discontinuous switches from one to two possible states may take place, are called *bifurcation points* (and the emergence of the discrete alternatives is called a *bifurcation*). Bifurcations occur wherever the system can be in qualitatively different states or stages. They are characteristic of qualitative change in development.

A final property of dynamic systems that is worth mentioning here is that they are often interlinked on all possible levels. For instance, the system's output may itself affect a control variable that in its turn governs the output. As a rule, the system and its environment stand in a relationship of mutuality: one affects the other and vice versa. This mutuality is often responsible for much of the non-linearity that is so characteristic of developing systems in general.

Systems, Environments and Self-Organization

We have seen that dynamic systems are (often, not always) characterized by an interesting property, self-organization. Lewis (1994; 1996), for instance, has presented self-organization as the hallmark of the dynamic systems approach to developmental processes. Cognitive structures come about as a result of self-organization; basic emotions are not innate but emerge as a result of early and rapid self-organizational processes (Camras, 2000). Although such processes involve extremely complicated self-organizational processes, which we still do not understand, the principle of self-organization itself is relatively simple to explain.

Let us take the case of language acquisition as a process that occurs between an infant and a competent speaker (note that the 'competent speaker' could also be the collection of all competent users of the language that effectively relate to the infant at issue). Define the infant as the system and, hence, the competent speaker as the system's active environment. There exists a constant flow from the environment to the system and vice versa. Since we are dealing with language acquisition, the flow consists, on the one hand, of the language by the mature speaker that is picked up by the child and, on the other hand, of the language by the child that is picked up by the mature speaker (or anyone else; note that what the child itself tells to others is also part of the linguistic input it gets; see Elbers' 1997 input-as-output thesis). These flows have a specific *structure* or *order* (in a system consisting of a steam engine and a heat source, the flow is one of thermal energy from heat source to engine, and the 'order' is the temperature). In the case of language, the structure or order of the environment-system flow (the language addressed to the child) is characterized by a grammar. A grammar is a set of specific, coherent rules necessary to explain the language as spoken to the mature speaker (a grammar is a formal description of the basic properties of a language; it is not a description of the internal mechanism that lets a speaker speak his mother tongue). The old – let us call it the pre-Chomskyan – view on language acquisition was that the environment-system language flow *overdetermines* the structure required to produce it (which is the grammar). By this we mean that the language contains more than sufficient information to reconstruct the grammar. Put differently, according to this view, the system – the language-learning child – receives the language flow (usually called the input) and this flow or input contains more than enough information for the child to reconstruct the grammar of the language. This situation basically complies with the second law of thermodynamics, also known as the entropic principle, which we discussed earlier. The law said that natural processes always show a decline in order or effectiveness. The thermal energy put in a steam engine is always more than the effective labor the engine produces. Similarly, when a message is transmitted, there is always a loss of information. Therefore, the message must be redundant, that is, contain more information than will effectively be retrieved by the receiver. That is, in order for the child to be able to construct the grammar from the input, that input must specify the grammar in a redundant (that is, overcomplete) way.

We have already seen however that Chomsky showed that language, as presented in the flow from adult to infant, *underdetermines* the structure required to produce it (the grammar). That is, the language contains *not enough* (instead of too much) of the information required to reconstruct the grammar. Because the child has not enough information to reconstruct the original grammar, the grammar that is actually constructed will be considerably poorer than the grammar of the adult. If we follow this line of reasoning and imagine the infant growing up to become an adult who addresses language to his or her offspring, the offspring will construct even poorer grammars (it goes without saying that children cannot look into the adults' brains and see the adults' grammars; they can only listen to what the adults are saying). In a few generations, language will be wiped off the surface of the earth. This is of course not what happens. In spite of the linguistic inflow underdetermining the grammar, the child nevertheless reconstructs the grammar required to produce the language. Put differently, the child produces a structure (grammar) that is richer or more complex than the inflow upon which that grammar was based. Thus, contrary to the entropic principle, an *increase in structure* has occurred. We can also say that the organization of the result (the reconstructed grammar) is of higher complexity than the organization of the inflow (the language addressed to the child).

An even more compelling example of spontaneous increase in structure is the emergence of an entirely new language, based on the rudiments of several different languages. For instance, on the basis of highly impoverished Pidgin languages spoken in communities of slaves or laborers speaking different languages, children have built complete and complex new languages – so-called Creoles – in just a few generations (Bickerton, 1991).

It is this increase in order (complexity or structure) that we call self-organization. Self-organization can vary from only a very little increment in the structure provided, to the building of very complex structures, such as bodies of organisms that are massively underdetermined by the information contained in the genetic code alone.

It is worthwhile pursuing the issue of language acquisition because it is related to a major theoretical discussion in developmental psychology. We have seen that, according to Chomsky's analysis, the language input was of lower complexity (showed less specificity) than the grammar produced or learned by the child. Chomsky and many others concluded that there is no known learning mechanism that can explain this miraculous increase in structure (which is right, there is no *learning* mechanism that can do the job). The conclusion must be, therefore, that what the language input lacks in structure must be supplied by some other source of information. The only known source of information, other than the environmental input, is the genome, the collection of human genes. The reasoning was that, since the observed spontaneous increase in structure is logically impossible, the fundamental properties of grammar (that are not contained in the input) must be innate, i.e. genetically determined. Fodor (1975) used a similar reasoning to prove the impossibility of development *à la* Piaget: it is logically impossible for a representation to produce a representation of higher order. Hence, it is impossible for a cognitive system in stage A to produce the more complex properties of the higher stage B.

The problem with this line of reasoning lies with the relationship between the premise and the conclusion. It is correct that there is no known learning mechanism that produces an increase of order relative to the input or inflow. But it does not follow that development or acquisition of structures like language must be based on a *learning* mechanism. We know that there exist many processes that spontaneously increase the order given. We call them self-organizational processes. It is highly likely that language acquisition, similar to many other processes that involve growth and development, is such a self-organizational process. It is true that we do not have even the faintest clue of how this process actually works (but we don't have the faintest clue of how grammar could ever become represented into the genome either). But this lack of understanding does not imply that such a process must therefore be logically impossible.

Dynamics and Self-Organization in Classic Theories

The issue discussed in the preceding section is of central importance to developmental theory building and far exceeds the limits of the example given, namely language development. The discussion focuses on the question of whether development is a process that occurs by design or by a different kind of mechanism that lies in the process of development itself. The view that development occurs by design could mean either of two things.

First, development could be the long-term effect of a process of instruction guided by the environment. What we call development, under this view, is the accumulation of learning processes that come in many different forms: reinforcement and operant learning, respondent learning, imitation, modeling, rehearsal, verbal instruction and so forth. The order and structure inside the developing organism are entirely defined by the order and structure as provided by the environment. In cultural environments, this is the structure of historically evolved skills, knowledge systems, science and so forth. In this view, development is the direct consequence of instruction and education and amounts to a process of acculturation. The internal mechanism needed for such a process of transmission to be successful is relatively simple. It is just a general association-storage-retrieval mechanism characteristic of information processing systems in the most general sense of that term.

The second view that favors the development-by-design explanation puts the prespecified order and structure not in the environment but in the organism itself, namely in the organism's biological make-up as specified by the genes. According to this view, the genes specify the consecutive steps taken by the developing organism. It goes without saying that the genetic design depends entirely on some specific environment to get its work done. However, the environment as such is vastly insufficient to specify the path of development. Such specification lies entirely in the genes.

The developmental issue that is at stake here is usually seen as a fight between two opposites, namely genes (or body) versus environment. The standard solution to the controversy is to admit that both aspects play a role. However, from a dynamic systems point of view, the controversy in fact does not lie between the genes approach and the environment approach, and the solution to whatever the controversy is, is not one of combining the two approaches. The real issue is between both the genes approach and the environment approach, as instances of the development-by-design position, versus a position that sees development as a self-organizing process. Self-organizing processes use whatever possibilities are offered by both genes (and body) and environment, but those possibilities come about as a result of the ongoing dynamic process.

One of the classic theories of development, that of Piaget, has taken a definite stance in this debate. Although Piaget's theory is often and superficially seen as taking an interactionist 'both-genes-and-environment' position, it really focuses on the design-versus-self-organization question. In Piaget's model, the inflow from the environment is entirely defined by the organism's internal structure, that is by its means and tools for taking this input. A similar physical event, like a yellow plastic block entering the visual field of a person, leads to entirely different experiences, depending on whether that person is an adult or a baby. For the baby, the experience is entirely sensorimotor: reaching towards, grasping and holding the object in a firm grip. For the adult, the experience is one of a geometric object that eventually fits in with a broader geometric structure (a wall of plastic bricks for instance). This act of assimilation, as it is called, brings about a complementary act, that of accommodation, which implies that the internal tools that tailor the experience are altered by the experience itself. The magnitude of this alteration, however, depends on the broader structure into which the assimilation is embedded. For instance, with babies that are on the verge of establishing differential grip patterns, the experience of grasping a plastic block may help the infant differentiate between grips suited for angular objects and those for rounded objects. With babies that already possess such grip patterns, the experience does nothing else than consolidate the already established pattern. This differential effect is not trivial, since it hints at another important aspect of Piaget's developmental theory, that of internal organization. Grip patterns, geometric forms or whatever the person is able to grasp literally or figuratively do not come as isolated properties, isolated tools in the cognitive toolbox. The tools are internally organized into higher-order structures. These structures are the result of internal, autoregulative processes that operate on the properties of the existing cognitive tools and on how they relate to the environment, in terms of assimilation and accommodation. This internal structuring is governed by a tendency towards internal stability, or, in Piaget's terms, equilibrium. This automatic striving towards equilibrium is an intrinsic property of complex organic structures. An unfortunate dog that has lost a paw in a car accident will, after recovery, spontaneously adapt its gait pattern to the number and position of the remaining paws in order to compensate for the loss. Along the same lines, an experience that does not fit in with the existing cognitive structure will either be transformed into one that does not contradict that structure or lead to a change in the structure itself, such that the experience is no longer contradictory (for instance, an experience of the result of action that contradicts the person's expectation of what that action should have brought about). It is important to note that the properties of the internal organization are defined by the organizational process itself and by the contents on which it operates. This will lead to a succession of basic structural organizations, which are better known in the form of Piaget's major stages (sensorimotor, pre-operational, concrete operational and formal operational). The order of those major structures also results from self-organization. For instance, as the sensorimotor organization collapses under the pressure of experiences that no longer fit in with that structure's limitations, a new structural organization emerges. By logical necessity, it must be the pre-operational organization. By 'logical necessity' is meant that, given the properties of the preceding structure (sensorimotor), the nature of the experiences that it brings about and the nature of the organizational processes that operate on cognitive contents, no other form of organization can emerge than the pre-operational organization.

Piaget's view of development as self-organizational rather than occurring by design is highly radical. It focuses primarily on the developing organism itself and sketches a form of self-organization that pervades all aspects of development and leads to global, overarching structures that characterize the child as being in a particular developmental stage (the stage characterized by a single, overarching cognitive structure). Having said this, I do not intend to claim that Piaget's is a dynamic systems theory of development *avant la lettre*. However, in its emphasis on development as a self-organizational process rather than a process-by-design, it does contain a core that is entirely consistent with current dynamic systems theorizing.

Some authors, also working in the dynamic systems approach, have pitted their dynamic theories against the theory of Piaget, thus implying that Piaget's opposes the major dynamic principles of development (Thelen & Smith, 1994; 1998). What these authors are attacking, however, is Piaget's (alleged) representationalism, that is, his idea that actions are based on internal representations and schemes. Those schemes act as if they were internal instructions, ready to be retrieved and used to guide actions. According to Thelen and Smith, action – motor, cognitive or whatever – is not based on prespecified instruction sets in the form of internal representations and schemes. Actions 'self-assemble' on-line as they call it, that is, the structure of an action results from the acting itself and from how the acting brings about changes in the environment. Development alters the conditions of such self-assembly in ways that are not currently understood (connectionist network models may provide reasonable explanations of what happens here). It should be noted, however, that a rich, incomplete and by itself also evolving theory such as Piaget's is not necessarily explicit about all its potential claims. A scheme such as Piaget's does not necessarily imply a form of representationalism that we have been accustomed to since the heyday of information processing theory and cognitive science. By its very nature, a sensorimotor scheme, like the grasping scheme, must be something that is entirely specified in sensorimotor terms and comes into existence only in a sensorimotor act. Identifying such schemes with the internal conditions – whatever they are – that make the self-assembly of grasping acts possible is not necessarily at odds with Piaget's notion of scheme.

Examples of Dynamic Systems Models of Development

Knowledge and Knowledge Development as Dynamic Processes in Real Time

Thelen and Smith (1994, 1998) have presented dynamic systems theory as a theory of development. A good example of their approach is their work on a phenomenon called the A-not-B error (Smith et al., 1999). When infants between 6 and 12 months of age watch an object being hidden under some cover and, after a short delay, are given the opportunity to recover it, they will reach to the place where they saw it hidden. After a few trials in which the object is hidden in one place (called A) the object is then hidden – while the infant watches – in some other place, for instance to the left of the first place (place B). Although the infant has seen the object hidden in place B, he or she will nevertheless reach for it in the original hiding place A. The A-not-B error is a step in the process of the development of the object concept. It has been introduced by Piaget and has been extensively studied ever since (Wellman, Cross, & Bartsch, 1987). Basically, what Thelen, Smith, and co-authors react to is that the A-not-B error shows the manifestation of a – still immature – internal representation of the notion of object. They criticize the widespread standard conviction that this representation is an internal symbolic structure that features in set of internal beliefs and instructions that is supposed to guide the infant's actions. The standard conviction is supposed to be like this. The infant has an immature object concept, which is an internal symbolic structure specifying the properties of objects in general. In infants between 6 and 12 months, the object representation is still tied to the infant's representation of the action it has performed with the object. Hence the infant believes that in order to retrieve the object, he or she must repeat the action that was successful in the first place. This internally represented belief is thought to be the causal impetus behind the action 'reach-towards-A'. In Thelen and Smith's dynamic systems view, however, knowledge – of the object concept, for instance – is not some internal symbolic structure that causally guides actions. Knowledge is a process. It is the result of the dynamic process of interaction between a specific context and a specific body (a body with a specific past and history). The process unfolds by the continuous transaction between the context and the body, and both body and context change during that interaction and by so doing provide new conditions for further steps in the process. Consolidated knowledge (as when we say that a 15-month-old child *has* an object concept) means that the contextual and individual conditions are such that the process has zoomed in on some stable, repetitive pattern (e.g. the infant reliably retrieves the object from place B).

According to the analysis of the A-not-B error provided by Thelen, Smith, and co-authors, the phenomenon is not about objects and object concept development, but about ‘the dynamics of goal-directed reaching in unskilled reachers when placed in a task that requires them to repeatedly reach to one location and then reach to another’ (Thelen & Smith, 1998, 613). In order to explain the error, one should realize that ‘activity at any moment will be shaped by the just previous activity at any level’ (1998, p. 613). I introduced the general idea that underlies this principle, that of iteration, in the description of the general properties of dynamic systems. In a task like this one, the internal neural coding of the preceding act of reaching still persists after the reach. In skilled reachers, this coding is sufficiently counteracted by a neural coding based on an act of visual attention to the new target (place B). Unskilled reachers need a strong visual attractor and also one that immediately precedes the reaching in order to be able to decouple the looking from the reaching. In the standard A-not-B task, however, the hiding places are not strongly visually distinct from one another, and nor is visual attention to the B-position drawn immediately before the reaching is made possible. The net result of all these conditions is this: with repeated reaching the infant builds up a strong temporary reaching attractor to place A, which implies that the attractor persists after the reaching is finished and thus influences any consequent reaching act; the visual saliency of the B-place is not enough to overcome the reach-to-A pattern and also not enough to decouple the looking (to B) from the reaching (to A). Consequently, the infant looks and reaches to A. If the infant’s attention is drawn to B just before reaching is allowed, the infant can decouple the reaching and the looking and then the looking provides a strong enough attractor to guide the reaching to the place he or she is looking at. That is of course also the place where the object was hidden, which means that the infant no longer makes the A-not-B error. However, if we were to conclude that the reaching is now governed by a new internal representation, that of an object whose existence is independent of its movements, we would have made a serious mistake. In other words, what we see is a temporal pattern that entirely depends on the way the components of that pattern interact in time, on how and when they occur in the first place. But the meaning of the components of the pattern entirely depends on the just preceding events and on the internal condition of the reacher. The latter is the long-term product of the reacher’s history, his or her preceding experiences with reaching, looking and acting. According to Thelen and Smith, the disappearance of the A-not-B error is primarily based on the emergence of self-locomotion (walking, crawling). Self-locomotion stimulates the decoupling of reaching and looking because it requires that goals (where one is going to) are specified more or less independently of what one is momentarily looking at or what one is momentarily doing.

An important point of Thelen and Smith’s dynamic systems theory of development is that they do not make a distinction between short-term and long-term effects of actions. Actions affect the actor during the action by changing the actor’s expectations, skills and so forth (or more precisely, whatever internal mechanism corresponds with what we are used to call expectations or skills, for that matter). The short-term effects accumulate, in some way or another, and so correspond with long-term changes we call ‘development’.

In summary, the appearance and disappearance of the A-not-B error has nothing to do with the emergence of an internal, symbolic representation of the object that guides the child’s activity. It is the result of the dynamic coupling of actions and perceptions in an ongoing stream of context- and self-dependent activity.

Concepts and Representations in a Dynamic Systems Framework

The cognitive revolution in psychology has brought the notions of concepts and representations as explanation of human symbolic action to the fore. Thelen and Smith’s dynamic systems approach to knowledge, however, lies in the tradition of non-symbolist approaches to the nature of cognition. They defend the position that concepts and representations do not function as mechanisms of human action and, hence, that they do not exist. However, concepts and representations and similar notions are indispensable in the description of complex dynamic processes such as human action and they are perfectly compatible with a dynamic systems view if correctly interpreted. In my view, the controversy deals with a distinction between the question ‘What is it *that* you know?’ and the question ‘What is it *how* we know?’, which are related to the distinction between order parameters and control parameters that will be explained later. The question ‘What is it *that* you know?’ can be answered by specifying the content or nature of a person’s knowledge in some symbolic form. For instance, if asked what an 18-month-old child knows when she solves a Piagetian object retrieval problem, we may answer that she has knowledge about the fundamental nature of objects, that she has an object concept. By so doing, we give a symbolic description of what it is that the infant knows. More precisely, we give a description of what it is that the infant *relates to* whenever he or she is reacting adequately to the object problems with which the infant is presented: we say that in his or her reaching actions the infant relates to *an object*. (This is not a trivial remark. For instance, when I mistakenly hold Mr X for Mr Y and address Mr X as if he were Mr Y, *I* am in fact relating to Mr Y, though for someone else, who knows Mr Y and Mr X, I am relating to Mr X. This kind of relationship is usually explained by referring to beliefs – I believe that Mr X is Mr Y – but it is not implied that a belief must be something separate from the action, something 'in my head' that is independent of my actually relating to the alleged Mr Y.)

However, the – somewhat oddly formulated – question 'What is it *how* we know?' is all too often answered by invoking the answer to the first question (what we know) as a causal mechanism. For instance, when we say that an infant has an object concept, we explain the infant's behavior with objects by assuming that the concept is some kind of behavior-producing engine inside the child. However, this solution amounts to a category mistake, but it is a mistake that seems difficult to avoid, given our tendency to view concepts, representations and so forth as causal, internal entities. The answer to how you know the object concept lies in a description of the actual mechanisms of your behavior. These mechanisms can take the form of connectionist-network-like brain structures, specific problem contexts, dynamic interactions between such contexts and acting persons and so forth. An important merit of Thelen and Smith's approach is that they have tried to show how knowledge is brought about in the actual, dynamic process of action, which is a process that changes the conditions under which such actions are possible (or impossible) and by so doing covers both developmental and action time.

The distinction between 'what is it that I know' and 'how is it that I know' has a counterpart in a distinction made in dynamic systems theory (usually in the approach known as synergetics) between so-called order parameters and control parameters. The order parameters describe the 'order' of the behavior of the process, that is, its structure. The control parameters describe those aspects that cause the process to behave as it does, that is, as described by the order parameters. In complex, meaning-laden behavior such as human action, the order parameters, specifying the structure or nature of the behavior, are described by referring to the nature of what it is that the acting person relates to. Thus, the complex action of an 18-month-old child correctly retrieving a hidden object from a hiding place and who is no longer fooled by the A-not-B phenomenon, is described by saying that the infant 'retrieves an object', hence, that the infant 'has an object concept'. Note that the notions of concept and conceive stem from the Latin *concipio*, which means to take hold of, take up, take in, take or receive. Thus, if we say that an infant has an object concept, we express the fact that the infant takes this-or-that particular entity as an object (and not as something else). The notions of object or object concept are in fact the order parameters of the infant's behavior: the myriad of components that make up the infant's actual, conscious perceptual-motor activity are 'summarized', so to speak, by referring to the fact that the infant relates to an object. This order parameter, however, is different from the control parameter (more exactly, the many control parameters) that causes the behavior to self-organize in a form that we characterize by the order parameter 'object (concept)'. Those control parameters unfold in the form of a complex time-dependent dynamics, as Smith et al. (1999) have shown in their analysis of the infant's A-not-B error. That is, the notion of object concept does not refer to some internal set of representations that cause the infant to correctly retrieve hidden objects or to avoid the A-not-B error. It is in this sense of the word that concepts do not exist, as Thelen and Smith would contend. A similar reasoning applies to the notion of representation, but it would lead too far to pursue this issue here. Finally, it is worth mentioning that the problems with regard to concepts and representations that dynamic systems theory runs into, and cognitive science too, for that matter, were already being extensively discussed by the phenomenological psychologists who were active around the middle of the twentieth century and whose basic inspiration goes back to the philosopher Franz Brentano (1838-1917).

The Dynamics of Mental and Behavioral Ecologies

The present author's work on dynamic systems models of development has been strongly influenced by models from ecological biology. Ecologists study and model the dynamics of ecosystems (Kingsland, 1995). An ecosystem is a distinguishable structure of components – animal and plant species embedded in an environment of physical living conditions – that interact with one another and by doing so alter their presence in the ecosystem. The basic alteration applies to the species' population sizes. Stable ecosystems entertain some kind of dynamic stability that conserves the global structure of the system (the species involved). Ecosystems are explicitly resource dependent. To a particular species, the sum of all the species to which it is functionally connected and the physical living conditions form that species' resources for maintained existence. The study of ecosystems is primarily concerned with the study of how the available resources contribute to the structure of the ecosystem in space and time.

I have argued that psychological systems, in the broadest sense of the word, comply with the general, abstract principles of ecosystems (van Geert, 1991; 1993; 1994). Let us take a child's cognition and language as our universe of discourse and consider the child's linguistic knowledge as the system we are interested in. This system can be divided into various subsystems, for instance, the child's phonological knowledge, knowledge of the lexicon, of syntax, of semantics and so on. Note that this subdivision is just a functional simplification, defining the levels at which we want to study the system at issue (see the section on defining a system). Note also that by defining the subsystems as separable components (lexicon, syntax, semantics) I make no claim whatsoever about those subsystems' underlying forms. The point is that, whatever such components really are in terms of the underlying mechanisms, they can be fruitfully and meaningfully treated as separable but interacting components.

Each component in such a system is further defined as something that is subject to growth (such a component is, somewhat trivially, called a *grower*). For instance, the lexicon (knowledge of words) begins somewhere and sometime with a minimal starting level (metaphorically speaking its germinal state) and grows towards some form of dynamic stability in adulthood (which lies probably around 200,000 lexical entries, i.e. basic words). Its growth is mathematically modeled by a simple equation, the logistic growth equation. This equation specifies growth as the joint product of the component's current growth level (thus obeying the iterative principle that a dynamic system is always governed by its just preceding state) and the available but limited external resources (the component's environment). Any additional component in the system (for instance syntax) to which the current component (the lexicon) is functionally connected forms part of that component's resource structure. The functional relationships are often symmetrical (syntax is a resource component of the lexicon, the lexicon is a resource component of syntax) and sometimes antagonistic (the lexicon positively affects the growth of early syntactic knowledge whereas early syntactic knowledge has a – temporary – negative effect on the growth of syntax).

A dynamic model – in this particular case a model of language growth – consists, first, of a specification of how the components involved in the system affect one another in terms of resource functions (e.g. L has a positive effect of magnitude *m* on S, S has an initial negative effect on L, and so on). Second, it specifies the initial conditions of each component, and third, the eventual conditional dependencies among the components (e.g. a specified minimal level of lexical knowledge is a precondition for the emergence of syntactic structures such as two-word sentences). Even a relatively small number of components easily results in a rather complicated web of relationships and mutual effects. The dynamics of such a web can only be understood by simulating its evolution under various conditions (e.g. stronger or weaker influences among the components involved, different timing of the emergence of components and so on). Simulation studies with those web structures demonstrate that they can spontaneously show qualitative and quantitative properties that are characteristic of development (van Geert, 1991; 1994). For instance, they settle into equilibria, show stepwise change, show stage-like coherence, run into temporary disorder, and so forth. The fact that such models show the required properties does not, of course, provide a proof that real development occurs along the models' principles. Part of the supporting evidence must come from studies that concentrate on specific aspects of the models and actively manipulate the model's control variables. Such manipulation is considerably more easily done in studies of motor development than in studies of language or cognition, for instance.

These models can also be fruitfully applied to specifying relationships between components at a much finer scale of detail. For instance, instead of specifying relations between lexicon, syntax, semantics and so forth, one may focus on specific syntactic rules or structures as components of a web of interactions. For instance, in a study on the emergence of verbs and prepositions, we studied various early preposition structures used by infants (e.g. N-Prep structures, as in 'doll in'). From the patterns of increase and decrease in the frequency with which these patterns occurred, we inferred a sequence of asymmetric relationships among those rules: preceding rules have a positive effect on the emergence of later rules, whereas later rules have a negative, i.e. competitive, effect on earlier ones and lead to the disappearance of the latter (see Figure 28.6).

Note that this pattern of relationships – a positive relationship from a developmentally earlier to a later component, and a negative relationship from a later to an earlier one – is probably quite universal in development. For instance, if applied to various stages of moral reasoning as described in the Kohlberg tradition, these relationships lead to the pattern of appearance and disappearance of moral reasoning styles found in a cross-sectional study by Colby et al. (1983; see van Geert, 1998b).

A disadvantage of these ecological models is that they provide no explanation for the actual emergence of new forms in development (new forms can easily be incorporated, but they are not explained). In an alternative to this ecological model, I introduced notions directly inspired by Piaget's assimilation-accommodation principles (van Geert, 1998a). In this model, environmental inflow is defined by the child's current state of development. The eventual progress the child makes, given this inflow, is based on that inflow and on the child's current developmental state. Stated in this general form, these principles are also present in Vygotsky's notion of the *zone of proximal development*, which implies, among other things, that children acquire new skills and knowledge when given help – by a more competent person – that fits in with their current developmental level. It turns out that a dynamic model based on these general and traditional developmental principles explains not only gradual change, but also discontinuous and stagewise change and changes in the variability of performance. In empirical studies, for instance, we found that day-to-day or week-to-

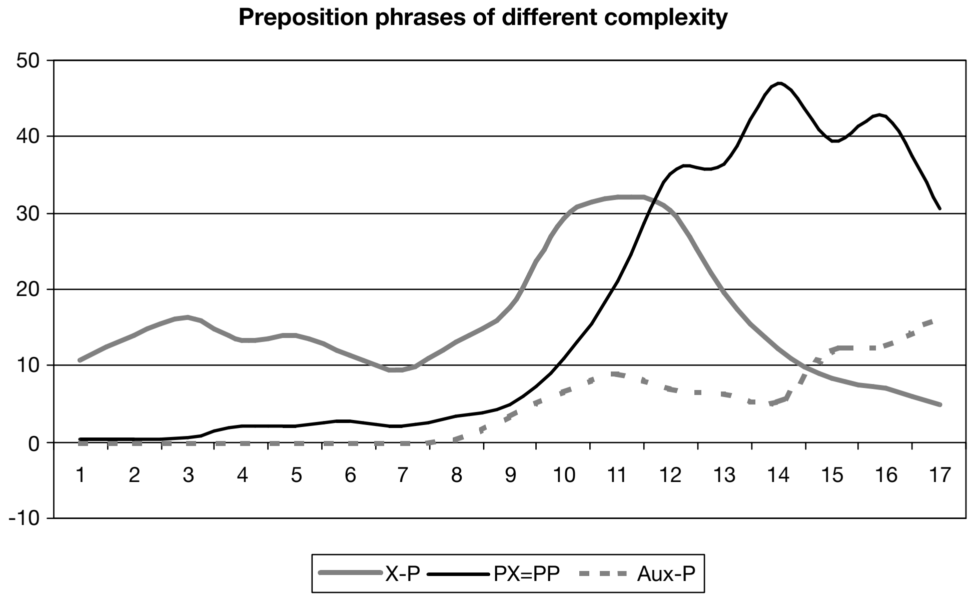

Figure 28.6 *Changes in the frequencies of utterances with prepositions. The utterances are ordered in terms of increasing syntactic complexity (X-P, PX=PP, Aux-P). The least complex form (XP) increases, then decreases as more complex forms emerge*

week variability in performance levels is quite considerable at the beginning of some developmental process and that the range of variation decreases as the process settles into some equilibrium level (de Weerth, van Geert, & Hoijtink, 1999). Note that the pattern of change in variation depends on the nature of the developmental process at issue: in some processes it must, by necessity, be small at the beginning and increase towards the end or increase just before the process jumps to a new equilibrium level (van der Maas & Molenaar, 1992).

Building Your Own Models: A Short Tutorial

Building dynamic models of developmental processes requires some special skills and practice, but those skills are not beyond the reach of anyone who has some experience with computers. There are several software packages on the market that are especially designed for systems modeling. A quick search across the Internet shows how many of such specialized packages have already been developed.[1] Several of those programs (such as Ithink and Modelmaker) provide relatively user-friendly interfaces, free demo-programs and worked examples. For the occasional model builder, they have the disadvantage that they are not always cheap and that they require considerable practice before they can be actually used. A good alternative – if one is aware of the unavoidable disadvantages – is the use of spreadsheet programs such as Excel. Such programs are widely distributed and considerably more versatile than many users think. With some extension of the skills that spreadsheet users mostly already have acquired, interesting demonstrations and explorations of dynamic interactions between variables are possible. Basically, the cells of a spreadsheet are predefined variables. We can specify an equation in each cell (variable) that refers to other variables. If we view every time step as a separate variable, we can use the spreadsheet program to build a (somewhat crude but essentially effective) model of dynamic systems processes. The idea of a time step needs a little more explanation – and caution. If we specify a simple conceptual model – for instance, that social experiences affect social knowledge in children – we are usually not explicit about whether the effect is continuous or discrete. In this particular example, it is likely to be discrete. Each time the child has a particular social experience, some of its social knowledge (whatever that is, in reality) is changed. In our model, each time step corresponds with a discrete event, namely the experience and its effect on knowledge. However, if we zoom in onto the event itself and imagine how a child perceives and evaluates the stream of actions in the social situation and acts him- or herself, the relationship between the variables at interest (the experiences and their effects) is more likely to take the form of a continuous stream of mutual effects. In a computer simulation, such continuous streams are always broken up into discrete steps, but in order to approach the continuous character of the events, the steps are made very small and specific mathematical algorithms are used to correct for the discretization. If one uses a spreadsheet program, continuous processes are best (but still somewhat crudely) approached by cutting the whole process into a (very) large number of steps. Discrete processes should be modeled by taking a number of steps that correspond with the number of discrete events one wishes to model. This, by the way, is just one example of the effect of modeling on theory building: when trying to construct the model, a whole series of decisions need to be made that require further theoretical and empirical analysis of the processes we model and, thus, potentially lead to a better understanding of such processes, even if the modeling itself proves in the end not very successful.

In line with the ecological approach I take with respect to developmental processes, I have often used an ecologically inspired model of increase or decrease in a variable, namely the logistic model (see the section on the dynamics of mental and behavioral ecologies). The logistic model, which describes the growth of populations, but which can also be applied to economic processes or to the growth of scientific publications, to name just two examples, views the increase in a variable as the effect of two sources of influence. The first is the variable itself, the second is the variable's environment, that is, the collection of influences outside the modeled variable (other variables 'inside' the subject and variables 'outside' the subject, i.e. his or her external environment). Take for instance a child's understanding of a simple arithmetic operation such as addition. The effect of information – feedback on an addition error made by the child, for instance – depends on the level of understanding already acquired by the child. The growth of the child's understanding of addition is based on – and therefore also limited by – the total set of resources that operate on that particular understanding. Those resources are internal (the child's knowledge and understanding of numbers, for instance) and external (the kind of help given by the environment, the opportunities given to the child for practicing addition, and so forth). According to the logistic model, any next level of some variable – for instance the child's understanding of addition - can be expressed in the form of the following mathematical equation:

$$L_{t}+_{\Delta t} = L_t + L_t \times \text{rate}_{\Delta t} \times (1 - L_t/K_t)$$

where $L_t+_{\Delta t}$ the next state of the variable and L_t is the preceding state; rate$_{\Delta t}$ is the growth rate that applies to the time interval Δt between the next and the preceding state; and K_t is the carrying capacity, which is the set of resources that apply to the variable at issue. This set of resources is expressed in the form of the equilibrium level K_t that the variable will eventually attain. This equation forms the expression of a simple but powerful dynamic process model. It is iterative in that every next step is the product of the preceding step (and something else). It is dynamic in that it models the change of the variable as a process that takes place over time. In the next section I shall give an example of how this model may be transformed into a spreadsheet model with which we can experiment.

Using Chopsticks To Eat Your Meals

Let us assume we are interested in the growth of a particular skill, namely the ability to eat with chopsticks. I shall assume that we have some kind of ruler against which we can measure an individual's chopsticks manipulation skill (note that we don't need to have such a measure or test in reality, it suffices that it makes sense to assume that such a ruler is available in principle).

First, we have to decide on some initial level, a 'seed' that must be bigger than 0. The 'seed' can be any arbitrarily small number (or a number that is based on empirical observations of initial state conditions). Let us assume that we arbitrarily set the initial level of chopstick manipulation to 1/100th of the average skilled chopstick user (we can try different initial states once we've set up the model). Let us also assume that the effect on the chopstick manipulation skill is based on discrete experiences, namely one meal a day (one meal a day eaten with chopsticks, that is). Assume further that we have observed that most children who begin with chopsticks at an early age take about three months to become really proficient with this equipment. Three months is rounded off to 100 days, which, given there is one practice event a day, gives a total of 100 practice events. Our model should therefore count about one hundred steps. We open a spreadsheet file and dedicate the first one hundred cells in the first column to our chopsticks model. Assuming that we set the level of skilled chopstick manipulation to 1, the initial level must, as agreed, be 0.01. We dedicate the first cell in the column to the initial state and write 0.01 in the first cell, A1 (this should sound quite familiar to spreadsheet users). A2 is the second practice event, A3 the third and so forth, up to A100. If the effect of practice is specified in the form of the logistic growth equation explained above, we can fill in the chopsticks manipulation skill level during the second event (A2, the second meal) by introducing the equation in cell A2, which, in spreadsheet format should look like this:[2]

= A1 + A1 * rate * (1 – A1/carrying capacity)

Recall that we decided that the level of a skilled chopsticks manipulator should be set to 1. That is, 1 is the level that will be achieved, given all the resources present in the environment (by resources I mean the subject's general motor level, muscle strength, eye-hand coordination, etc. in addition to teaching, examples and guidance with regard to chopsticks manipulation by the more experienced users in the environment). Thus, the A1 variable at the end of the equation is divided by the carrying capacity, which is 1, and this division can, of course, be omitted from the equation. Each of the remaining 98 steps in our model must refer to its predecessor (the preceding step) and calculate the level achieved in that step on the basis of the level achieved in the preceding step. In order to accomplish this, we simply copy the content of cell A2 to all the remaining cells in the column (a spreadsheet copy command will automatically make the correct reference of every next cell to its preceding cell). We have now completed our first dynamic model in spreadsheet format. If we graph the data in the column comprising the 100 steps (which is very easily accomplished in a spreadsheet program), we will see that chopsticks manipulation increases in the form of an S-shaped curve, provided the growth rate is not too small – and not too big either. If we set the growth rate to 2.85 for instance (which means that the change per event equals an almost threefold increase of the level, damping factors not taken into account, which is indeed very much), we see that the process turns into a chaotic oscillation. Although it is interesting to see that a change in one parameter can cause a qualitative change in the growth pattern (from a smooth to a chaotic process), there is no reasonable conceptual interpretation for such a high growth rate – and its effect – in the case of a motor skill such as chopsticks use. Thus, the mathematical possibility of a chaotic oscillation does not fit in with the nature of the process we are currently modeling (it may fit in with other processes, though).

Let us now extend the model by assuming that a person can lose his or her skill if the manipulation of the chopsticks is not sufficiently practiced. This is basically what occurs with an occasional visitor of oriental restaurants, who uses chopsticks only once in a while. We use the column right of the skill level column for specifying randomized intervals between events, i.e. between meals in which chopsticks are used. The equation for a randomized interval is as follows:[3]

1 + Int(Rand()* length)

If length is the maximal number of days between meals in which chopsticks are used, the equation produces a random number of days between 1 and the value of 'length'. This random period differs for each step in the process.

We assume that the skill level decays if it is not practiced. The decay is proportional to the level already attained, the length of the period without practice and the decay parameter. Our dynamic model is now expressed in the form of a more extensive equation, namely:

A1 + A1 * rate * (1 – A1/carrying capacity)
– decay * B1 * A1

Assuming that we have defined the parameter name 'decay', the additional part of the equation refers to the level already attained (which, for the second step, is the level attained in the first step, which is in cell A1) and to the number of days between the first and second meal with chopsticks, a value that can be found in the cell right of cell A1, namely cell B1. We copy the new formula and the random period formula to the 98 remaining cells in columns A and B. Each time we let the program recalculate the process, we will calculate new randomized periods and thus find different curves for our chopsticks skill level. With a little experimenting, we will discover an interesting property of our model, namely that the average period between chopsticks meals determines the (approximate) equilibrium level of the chopsticks manipulation skill. That is, the average period between meals turns out to be part of the set of resources and thus determines the equilibrium level of the skill. This is an interesting discovery (although it is a fact that can be mathematically inferred from our equations). Before trying the model out we would probably have thought that the practice intervals would result in a decrease of the learning speed, i.e. that it would take longer before the maximal level is attained. This little example shows that building dynamic models – however simple – and studying their properties based on varying the values of the parameters, may indeed lead to a better understanding of the properties of our models (Figure 28.7).

The preceding example had no other function than to show how a simple model could be built and implemented in the form of a spreadsheet program. Simple though it is, its basic principles can be applied to a host of more realistic examples, such as the growth of the lexicon, the growth of the use of syntactic structures and categories in language, the growth of cognitive skills and so forth.

A Model of Hierarchically Connected Growers

Some time ago, Fischer (see Chapter 19 in this volume) and myself cooperated on an attempt to build a dynamic model of Fischer's skill theory (Fischer, 1980). Skill theory describes domain-specific development in the form of a series of major stages or 'tiers', subdivided into substages or levels. The series forms a hierarchical structure. Each lower level is a precursor to its higher-level successor and

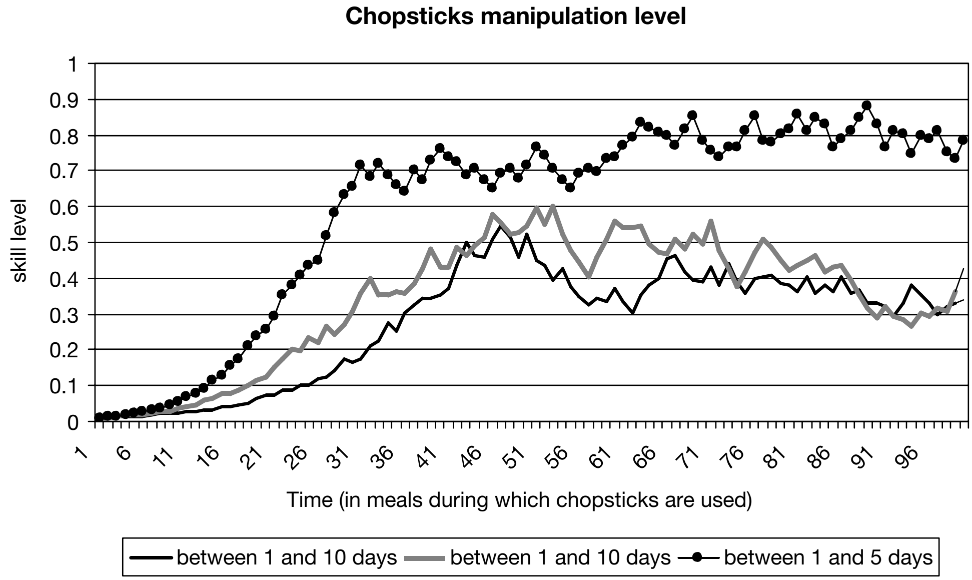

Figure 28.7 *The growth of chopsticks manipulation skill. The model is based on a random factor-driven logistic model. The increase or decrease of the skill level is based on discrete learning events, namely meals eaten with chopsticks. The intervals between the meals are also randomized, with a maximum interval of ten and five days respectively. The five day maximum results in a higher and more constant level of skill than the ten day maximum, all other factors being equal (except for the random variations in the parameters). Random variations in the parameters may cause differences in the rate of growth (see the graphs based on the ten day maximum)*

is integrated into the higher level as soon as the latter has emerged. A simple example of three such levels concerns the understanding of an arithmetic operation such as addition. Level *A* consists of the ability to solve simple addition problems. Level *B* involves an abstract understanding of the addition operation as a combination of smaller units into a larger one. Level *C* involves the abstract under standing of a relationship between arithmetic operations, for instance addition and abstraction. Each level can be conceived of as a 'grower'. A grower is a variable that starts at an initial level and that increases by way of a process described earlier in the form of the logistic growth equation. A model of the three levels described above would consist of three logistic growth equations, one for each level of addition understanding. According to Fischer's theory, the three growers are hierarchically connected. What does that mean in terms of a mathematical relationship between the three equations? First, the connection applies from the lower to the higher level and concerns a conditional relationship. The higher level cannot get off the ground as long as the lower level – the condition or precursor – has not yet reached some minimal, conditional level (i.e. abstract understanding of the addition operation is not possible without a reasonably developed skill in solving simple addition problems). Recall that the logistic growth equation has the following form (it has been written in the form of an equation for level *B*):

$$B_{t+t} = B_t + B_t \times \text{rate}_{\Delta t} \times (1 - B_t / K_t)$$

We know that *B* cannot get off the ground as long as the level of its predecessor, grower *A*, stays beneath a conditional level, which we will set to A_c (determining this level would normally be a matter of empirical research). We now introduce a parameter C_B and alter the above equation as follows

$$B_{t+\Delta t} = B_t + [B_t \times \text{rate}_{\Delta t} \times (1 - B_t/K_t)] \times C_B$$

C_B is a parameter with only two possible values, 0 and 1. The equation for C_B is as follows:

$$C_B = 1 \quad \text{if} \quad A_t \geq A_c$$
$$C_B = 0 \quad \text{if} \quad A_t < A_c$$

for A_c, the conditional value of the variable *A*. Note that equations like these can be very easily written down in the form of a spreadsheet model. Assuming that the values of variable *A* are in the A-column and those of variable B in the B-column, we use the C-column to calculate the C_B value. We select the second cell of the column, i.e. the first row after the row that contains the initial values of A and B. Assuming this is cell C2, we write down the following equation:

= If (A1 = >0.8,1,0)

This equation should be read as follows: if the value in cell A1 is equal to or bigger than 0.8 (the value that we have taken as the conditional level necessary for variable B to start growing), the value in cell C2 is 1; if the condition is not fulfilled, the value in cell C2 is 0. A different, more abstract way of reading this equation is IF condition A1=>0.8 is true, THEN 1, ELSE 0. In cell B2 (which I assume to be the second step in the calculation of the variable B), we write down the following equation:

$$\mathrm{B2} = \mathrm{B1} + [\mathrm{B1} \times \mathrm{rate}_B \times (1 - \mathrm{B1}/K_B)]\mathrm{C2}$$

(recall that B2, B1 and C2 refer to the values in the cells B2, B1 and C2 respectively). It is easy to see that as long as the values in the A-column (the A variable) remain below 0.8, the corresponding value in the C-column remains 0. Since $[\mathrm{B1} \times \mathrm{rate}_B \times (1 - \mathrm{B1}/K_B)]$ 0 is of course 0, the growth equation amounts to $B_{n+1} = B_n + 0$, which means that every next cell is equal to its predecessor, which simply means that the value of B does not change. As soon as the A value is equal to 0.8 (or any other value we find appropriate), the value in the C-column turns into 1, and the growth equation starts to take effect. Since the conditional relation holds between any level and its successor, it also holds between growers B and C. A similar set of equations can be set up, applying to columns B and C respectively.

We have now modeled the first part of our theory, namely the notion of a conditional or precursor relationship. The second part seems a little bit more complicated. What could we possibly mean by saying that the less complex level is incorporated in the more complex one, or that the less complex level is integrated or incorporated in the more complex one? Since we are dealing with the quantitative aspect of the variables only (we are modeling their level) we must translate this idea of integration into a quantitative relationship. When we say that A, the simple addition skills, have been integrated into B, the more complex level of understanding of what addition actually means, we intend to say that A has changed, that it has become an expression of B rather than the old, limited understanding as expressed in the original skill of solving addition problems. We may argue that this change in the nature of A should lead to an improvement in the expression of A, or, stated more simply, that an understanding of what addition really means should allow a child to solve addition problems (the A skill) more adequately, with fewer errors, or that more complicated addition problems can be solved. Note that this is the theory, not necessarily the empirical reality. But what we are trying to model here is the mechanism or the relationships as postulated by the theory. Whether or not these relationships also cover reality remains to be seen, but that problem is not at stake here. If it is indeed so that an integration of A into B should lead to an improvement in A (this is what the theory says), we could increase the level of A as a function of the level of B. Thus, the higher the level of B, the higher the level of A, or A grows as a function of the growth of B. This relationship can be expressed in the following form:

$$\Delta A = A_t \times B_t \times \mathrm{support}_{B\mathrm{to}A}$$

which should be read as follows: part of the increase of A depends on the level of A at time t, the level of B at time t and a factor that expresses the degree of support from B to A.

Now the full equation for the variable A is as follows:

$$A_{t+1} = A_t + A_t \times \mathrm{rate} \times (1 - A_t/\mathrm{carrying\ capacity}_A) + A_t \times B_t \times \mathrm{support}_{B\mathrm{to}A}$$

By now, it shouldn't be too difficult to transform this equation into a spreadsheet model. Note that with the equations for the variables B and C the increase component (the part of the equation after the first A_t at the right-hand side of the formula) is multiplied by the conditional factor C_B (or C_c). The resulting graphs show a series of three 'stages'. The first consists of level A only, the second witnesses the emergence of B and an increase in A, and the third involves the emergence of C and an increase in the levels of B and C (Figure 28.8).

Our model has, of course, severe limitations. It confines itself to the quantitative aspects of growth and it does not explain why B emerges, given A (we only show that B will not get off the ground as long as A stays beneath some preset value, but we haven't explained why it is B that grows, and not C or D or E . . .). However, in all its simplicity, it nevertheless demonstrates some interesting properties of the developmental dynamics. For instance, it shows how patterns such as stepwise growth emerge from the nature and the interaction. The timing of growth spurts (and related phenomena such as dips) is not preprogrammed by some internal alarm clock but depends entirely on interactions between variables that are there all the time. Models such as these also allow the researcher to experiment with many different parameter settings in order to investigate the range of possible outcomes that the model allows for. It is usually hardly possible to infer this range of possible patterns on the basis of the conceptual model alone. One needs to turn one's conceptual models of processes into calculation procedures –

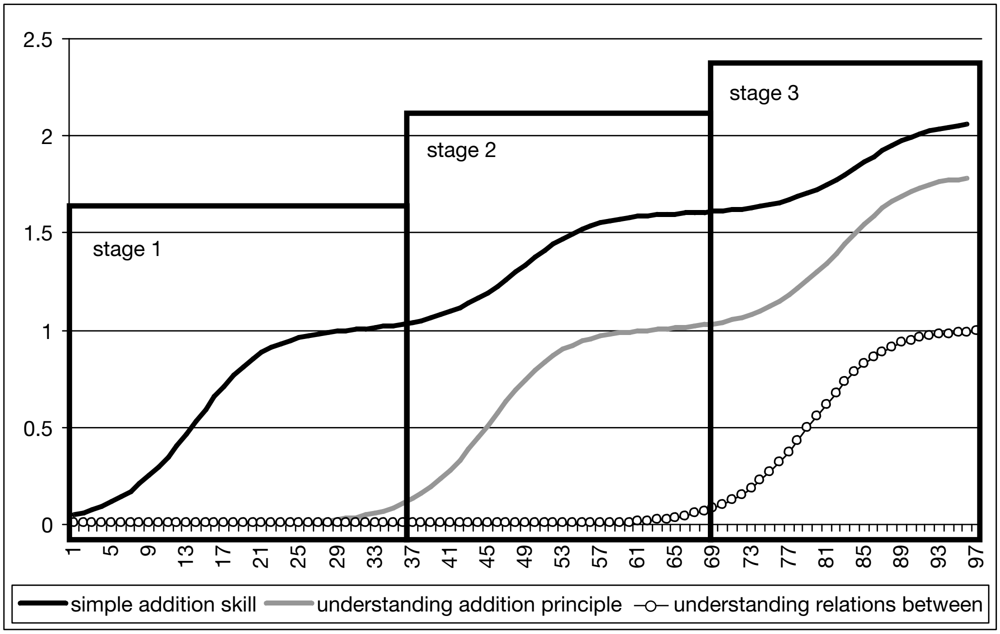

Figure 28.8 *A simulation of three hierarchically coupled growers based on Fischer's (1980) skill model. The first grower is a precursor to the second, and the second to the third. The first grower profits from the growth of the second, and the second profits from the third. The 'profit' is the effect of hierarchical integration of the earlier grower into the more advanced grower (its successor). The result is a series of stepwise increases that resemble a three-stage growth model*

i.e. into dynamic models – in order to obtain an idea about the model's inherent possibilities. If researchers want to use the models to predict or explain empirical phenomena, it is of utmost importance that they know what their models are capable of. In this way, the building of dynamic models is an important, if not essential, step between conceptual theory formation and the empirical testing of the theory. In this regard, I have compared the building of dynamic models with doing experimental theoretical psychology (van Geert, 1994). It is theoretical psychology in that it concerns the researcher's conceptual models, but it is experimental in the sense that it consists of experimenting with possible parameter settings in order to find out what the model will do.

Summary and Conclusion

The notion of development, as an unfolding of inherent properties and a tendency towards increasing order and structure, plays an important role in everyday discourse. As a scientific concept, it was largely discarded by developments in physics and biology, which emphasized that order does not spontaneously increase but only decreases and that evolution does not involve an intrinsic tendency towards more complexity and 'higher' forms. Although these conclusions still hold, they have been explicitly modified and amended by developments in dynamic systems theory, which studied the properties of processes where order and structure are spontaneously increased and gave a new and considerably more exact meaning to the notion of inherent property. With its emphasis on the central importance of interaction in real time, dynamic systems theory forms a natural framework for the study of development. However, dynamic systems theory is not a single theory but a general approach, with many different possibilities. After an overview of the many faces of dynamic systems theory, we proceeded with a discussion of some of its applications in the field of developmental psychology. These applications are still haunted by a host of problems of a methodological, conceptual, modeling and empirical nature. Development itself, however, is a difficult notion and in general a very tough nut to crack. In fact, it is considerably easier to altogether abandon the notion of development and explain developmental phenomena by reference to internal programs or instruction sets, such as the genetic code. If we take development seriously, however, an approach like dynamic systems or something along similar lines will probably be the only way out, however premature and incomplete such an approach at present may be.

Notes

1 Try for instance http://www.acs.ilstu.edu/faculty/wjyurci/nsfteachsim/simsoftware.html; http://www.idsia.ch/~andrea/simtools.html.
2 I assume that the spreadsheet user has already defined the parameter 'rate' as a name in the spreadsheet; Excel users do this by clicking Insert/Name from the menu.
3 This is how Excel would specify that equation; in another spreadsheet, the equation will probably be a little different from this one.

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