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# Explaining after by before: Basic aspects of a dynamic systems approach to the study of development

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**Abstract**

The basic properties of a dynamic systems approach of development are illustrated by contrasting two simple equations. One, $y_{t+1} = f(y_t)$, is characteristic of dynamic systems models. The other, $y_i = f(x_i)$, refers to what, for the sake of simplicity, is referred to as the standard developmental approach. We give illustrations from cognitive, language and social development to show the characteristic differences of these two types of models and show their complementarity. The article further compares the "Bloomington" with the "Groningen" approach to dynamic systems theorizing in developmental psychology. It continues with a discussion of two important questions. One involves the issue of measurement and the nature of developmental variables from the viewpoint of dynamic systems. The second concerns the question of short- and long-term time scales in developmental models, which is discussed on the basis of an example, namely dyadic interaction of young children in the context of different social statuses.


*Keywords:* Dynamic system theory; Mathematical models; Vocabulary development; Dyadic interaction; Social development; Measurement; Time scales

## Dynamic systems gets you into problems…

Being a developmental psychologist and applying dynamic systems theory is almost like begging for trouble. A quick look through handbooks on dynamic systems theory (for

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instance Jackson, 1991; Katok & Hasselblatt, 2005) reveals an amassment of abstract terms and mathematical equations most of which are simply not accessible for the mathematically untrained reader, which the developmental psychologist is likely to be. It gets worse if instead of the handbooks the real mathematicians working in this field are consulted. As a rule, the mathematician will treat the developmentalist's dynamic models the same way as a kindergarten teacher evaluates a toddler's proudly made scribbles, knowing that the child in question deserves encouragement and that the way leading to a decent drawing of say, a horse, is still extremely long and arduous. And it also does not get really better if the developmental psychologist decides to turn to books discussing applications of dynamic systems to fields he or she feels at least a little bit accustomed with, such as biology or economics (see for instance Hofbauer & Sigmund, 1988; Murray, 1989; Ruth & Hannon, 2004). The problem now is not only the mathematics, but also the biologist's or economist's data, that seem to be so much more comprehensive and applicable than those normally available in studies of psychological development. And this is only one side of the problem. At the other side, in developmental psychology itself, dynamic systems is not really a very big issue. Child Development, for instance, has published 5 articles that have "dynamic(al) system(s)" in the title between 2005 and 1990. In the same period, Developmental Psychology published only such article with "dynamic(al) systems" in the title and so did Development and Psychopathology. Developmental Science published a special issue on the link between connectionism and dynamic systems (September 2003). Other examples are the issue of the Journal of Abnormal Child Psychology of December 2004, devoted to advances in process and dynamic system analysis of social interaction and the development of antisocial behavior, a special issue of the Journal of Experimental Child Psychology of October 1994 on dynamic modeling of cognitive development, and the current issue of Developmental Review. It seems as if the relative lack of publications applying dynamic systems theory to developmental processes has everything to do with the first kind of problem, namely the demands of dynamic systems theory both in terms of mathematics and formalization and of data collection.

Those of us working in the dynamic systems tradition have tried to overcome these problems in diverse ways. An important solution to the aforementioned problems originated from the work of Esther Thelen (we will call it the "Bloomington version"), represented by researchers such as Thelen, Smith, Spencer, Schöner and several others. Thelen and Smith (Thelen & Smith, 1994) took a number of general qualitative properties from dynamic systems theory—self-organization, complexity, attractors, phase shifts—and applied these notions to various developmental phenomena. A comparable approach, that is, via the qualitative properties, is represented in the work of Marc Lewis and co-workers (see for instance Lewis, 2000, 2004; Lewis, Lamey, & Douglas, 1999). Perhaps the smartest "move" of the Bloomington approach is that it has turned dynamic systems theory into a specific theory of development. Although the Thelen and Smith, 1994 volume is titled "A dynamic systems approach ...," it basically presents a particular theory of development as a process that takes place in real time and real action and that involves a close loop of interaction between an acting person and an acted-upon environment. It is highly empirically oriented and has led to a number of original experimental studies. The recent assimilation of dynamic field theory into Thelen and Smith's dynamic systems theory has opened the possibility of rigorous mathematical modeling of a kind similar to that discussed in the general dynamic systems literature(Erlhagen & Schöner, 2002; Schutte & Spencer, 2002). However, according to an aphorism of the world-famous Dutch soccer player Cruijff, "elk-voordeel-heb-zijn-nadeel," which, to retain the somewhat peculiar Dutch, must be translated as "each-advantage-haves-its-disadvantage." The coupling of development to real, spatiotemporal action makes clear that all actions, cognitions, evaluations, and so forth are embodied phenomena and are constrained and governed by the conditions of this embodiment (see also Clark, 1997). It also allows for dynamic modeling based on the physical parameters of space and time. However, by doing so, treatment of more "psychological" phenomena such as representations or meanings (whatever their nature might be) becomes exceedingly difficult. Thus, the application of the theory tends to limit itself to—or particularly focus on—spatiotemporal action at relatively early ages.

A different stance has been taken by the first author of this article (see for instance van Geert, 1991, 1994a, 1994b, 1995a, 1995b, 1997,1998, 2003). The main idea is that a dynamic systems approach of development lies in the application of the most fundamental—and simplest—representation of a dynamic system to developmental phenomena.

The main goal of the present article is first, to explain what this idea entails and to put it into the perspective of the work of others, such as the more "standard" developmental research on the one hand and Esther Thelen's approach on the other hand. The second goal is to hopefully falsify the title of this introductory section by showing that an application of this idea might contribute to a better understanding of developmental processes, without delving into the peculiarities of formal dynamic systems theory. To accomplish these goals, we will first discuss the basic form of a dynamic systems model in the form of a simple equation and contrast it with the basic equation that underlies most of the standard work in developmental research ("standard" is not in any way intended to sound pejoratively here). This is followed by a discussion of what it is that dynamic systems models of development apply to. The discussion will involve further reflection on the Bloomington approach, on the issue of developmental scales and rulers and on the links between developmental time scales and the models of those time scales.

## The study of development: Two basic equations

The two basic equations that will be discussed in this section reflect an almost paradigmatical difference in the approach to development. In this light, it is worthwhile to give some thought to the original meaning of the word "development." Etymologically, the English word *development* stems from the Old French *desvoloper*, which means "to unwrap." The German and Dutch words *Entwicklung* and *ontwikkeling* are literal translations of that term. The word is also related to the Latin *evolutio* (to unroll) and *explicatio* (to unfold). Explicatio is related to *to explain*, which therefore bears an interesting relationship to the word *development* (Thomae, 1959; Trautner, 1978). The etymological origins of development and related words suggest a process that is strongly driven by inherent properties, by something that is already there at the beginning and needs to be unpacked. The unwrapping carries the connotation of some sort of cyclical, continuous process where the wraps are taken off.

It is interesting to note that many classical theories of development, more precisely those that represented an explicitly organismic approach, remained quite close to the metaphor of an unfolding structure. In Piaget's theory, for instance, development carries the implicit notion of unfolding, which is related to the notion of transformation and development through qualitatively different stages. By way of a metaphor, one may think of a stage transition as what happens to a piece of paper that has been folded in some complicated way and that takes a different form each time one of the folds is opened for a structural analysis of Piaget's theory and of various other classical theorists, see (van Geert, 1987a, 1987b, 1987c, 1987d; van Geert, 1988).

Classical theories tended to view development in a retrospective way. That is, they started from a conceptualization of the main properties of a mature state (e.g., of cognitive development). Given these properties, the researcher can arrive at the properties of the preceding developmental states in an almost logical fashion by a stepwise elimination of the main properties that characterize the mature state. In combination with the notion of unfolding, this classical approach suggests that development is driven by some sort of inner logic or a sort of inevitability where a developmental process realizes its internal potentialities. As we shall see in the following sections, something of this notion of an inner logic is present in the dynamic systems approach to development, but considerably less so in the more standard approach. We will first discuss the dynamic systems approach by elucidating its basic equation.

## The basic equation of dynamic systems

The CRC Concise Encyclopedia of Mathematics (Weisstein, 1999) provides a particularly helpful—and remarkably frugal—definition of a dynamic system. It defines it as "... a means of describing how one state develops into another state over the course of time" (Weisstein, 1999, p. 501) . Thus, if $y_t$ is a specification of a "state" of a variable $y$ at time $t$, a dynamic model takes the form

$$y_{t+1} = f(y_t), \quad (1)$$

which should be read as "the value of $y$ at time $t + 1$ is a function "$f$" of the value of $y$ at time $t$."[1] A state is described by the value of a variable (or several variables, for that matter). The change in the value is a function of the variable's current value. Thus, a dynamic systems model of cognitive development (if any such model exists) is an explicit prescription (the $f$ in the equation) of how the current state of cognitive development evolves into another state, at some later moment. That is, the next state is a transformation of the current state, according to some explicit model or set of rules.

The basic equation is "recursive," or "iterative." That is, it transforms $y_t$ into $y_{t+1}$, $y_{t+1}$ into $y_{t+2}$, $y_{t+2}$ into $y_{t+3}$ and so on. The series of successive $y$'s forms the description of a process. In many cases, the change in one variable will be related to the change in another variable, and vice versa. For instance, if we plot the change in intensity of emotional expression over the short-term course of a social interaction between two children, the intensity of the first child's expression is influenced by the intensity of the second child's expression and vice versa; that is they are dynamically related (Steenbeek & van Geert, 2005a). Hence, the equation can be written as

$$\begin{aligned} y_{t+1} &= f(y_t, z_t), \\ z_{t+1} &= g(z_t, yt) \end{aligned} \quad (2)$$

for $y$ emotional expression in the first and $z$ emotional expression in the second child; the function $f$ specifies how the next state of the first child's emotional expressions depends on

[1] The common mathematical notation of Eq. (1) would use $x$'s instead of $y$'s. However, psychologists are used to refer to the variable they want to explain—in a regression model, for instance—by means of the $y$-symbol. We comply with this custom by using $y$ to refer also to variables featuring in a dynamic equation.

its preceding state and the state of the second child; $g$ specifies how this happens for the second child. This coupling of equations can be extended towards any desired level of complexity.

In our view, any model that complies with this basic definition is a dynamic systems model, which implies that the term "dynamic systems" is not confined to a particular theory (such as Thelen and Smith's theory; see Thelen & Smith, 1994). There exists a considerable variety of models that comply with this basic equation, including Markov chain models, simulation models, difference and differential equations (over time), verbally formulated models and so forth. The abovementioned equations distinguish a dynamic systems approach to development from a non-dynamic systems one.

The equations refer to states ($y, z, \ldots$) and functions ($f, g, \ldots$) Hence, any application to real development requires answering the following questions. First, what is the nature and what are the properties of the possible states of development, i.e., of the $y$'s and $z$'s. The second concern is of course about the nature of the function that transforms the states, i.e., the $f$'s and $g$'s. Since we have just reflected on the classical notion and theories of development, let us pursue that line and discuss the example of cognitive development in Piaget's theory.

*Example: Piaget and the issue of cognitive development*

Piaget's famous stage theory is probably just a particular way to describe the possible "states" of cognitive development. His theory of adaptation, including the mechanisms of assimilation and accommodation, is likely to represent the sort of function or rule, similar to the $f$ in the basic dynamic equation, that describes how one state results from applying that function to an earlier state. Although the theory of stages is commonly seen as Piaget's major contribution to developmental science, Piaget himself was remarkably loose with regard to the question of how many such stages there were. In 1947 there were five stages, in 1950 there were six substages in the first, sensorimotor stage, in 1955 every stage was conceived of as containing two substages, in 1970 there were three main stages each with two substages, and again in 1970, four stages were distinguished (Piaget, 1947, 1950, 1955, 1970a, 1970b). All this is not mentioned with the intention of making fun of Piaget. To the contrary, it illustrates that in opposition to the unwarranted emphasis that current handbooks put on a model of stages, Piaget's main concern was to focus on the way in which cognition changed, that is, on the characteristic, developmentally relevant distinctions between levels or "states" of cognition. In fact, his theory gives an answer to the question how many developmentally relevant "$y$'s" one can discern, which is a major point if one wishes to arrive at a specification of the "$f$" in the equation. Without too much of an exaggeration, we can say that Piaget's theory is basically of the form

$$\text{later state of cognition} = \textit{mechanism of adaptation}\ (\text{earlier state of cognition})$$

which one should read as a more detailed, verbally formulated expression for Eq. (1), $y_{t+1} = f(y_t)$. In this context, it is interesting to observe that neo-Piagetian theorists have embraced dynamic models, growth models in particular (Case & Okamoto, 1996; Demetriou, Christou, Spanoudis, & Platsidou, 2002; Fischer & Bidell, 2005; Fischer & Rose, 1994)

Thus, the complex, biologically inspired function that Piaget called (the mechanism of) "adaptation" operates on some particular current state of cognition that is acted out in a particular environment, and transforms this current state into some later state. The final point of the process is what Piaget called "equilibrium," which is not a state of rest but a state of active self-reproduction. That is, there exists a state (the state of formal operational thought) such that, if adaptation operates on that state, it produces a new state with the same major properties as that state. Maybe this is a somewhat awkward way of expressing the nature of a final state of development, but it points to a major fact of dynamic systems, namely the existence of stable states ("attractors" as they are sometimes called) that are just as well the product of the dynamics as the transient states where change occurs.

It is important that, if one is interested in the question of how a current state is transformed into a later state, it does not really matter very much where that process of transformation is studied. That is, one can just as well observe one's own three children Jacqueline, Lucienne and Laurent as anyone else's children. To a researcher accustomed to the standard statistical approach to studying development, this may sound like chutzpah. How can one obtain valid information about development by studying a very unrepresentative sample of three children of a Swiss psychology professor? The point is that the question of how one state is transformed into another and by what mechanism this occurs can only be discovered by studying an actual developmental process in its entirety, and this must logically begin by one single child. It cannot be discovered by reflecting on the statistical distribution of a particular state or states across a population (and this is likely to be so, even in the case of a population studied longitudinally in the standard sense, that is with a few repeated measurements separated by relatively long intervals; see further in this article). It goes without saying that the few subjects that one studies intensively over time should be exemplary with regard to the class to which they belong (e.g., young children, who do not suffer from particular handicaps or who do not live in harsh, impoverished environments). The sample need not necessarily be statistically representative (representativeness becomes an issue only if many such individual studies become available). More precisely, the test of the prediction that $f(y_t)$ produces $y_{t+1}$ depends on whether $y_{t+1}$ appears after $y_t$ has occurred, and not on the distribution of either $y_t$ or $y_{t+1}$ in the population (Molenaar, 2004).

### *The basic equation of standard developmental studies*

It is interesting to note that in one of the first quantitative and statistical studies devoted to Piaget (Elkind, 1961a, 1961b) the focus was explicitly on whether Piaget was correct with regard to the ages and order of the children's discovery of various types of conservation. With these two little studies, Elkind has set the stage—implicitly or explicitly—for many thousands of quantitative studies to come in the years after him. Whatever their complexity in terms of models tested, they share with these two short studies an important, basic equation, just as basic as the basic equation of dynamic systems we presented in the preceding section. To make it clear, let's go back to an example from Piaget. Conservation understanding is just one of those aspects of cognitive development that Piaget's theory tried to explain by saying "specify a state of conservation understanding—e.g., non-conservation, conservation of mass, etc.—and my theory will give you a state of conservation understanding that will succeed the one specified." This type of statement can be expressed in the language of mathematical functions as the now familiar Eq. (1), $y_{t+1} = f(y_t)$, which is the elementary format of a dynamic systems function. What Elkind says—and many others in many different variations after him—is different: "specify an age and my theory will give you a state of conservation understanding" (with a certain amount of error, but this is also the case with the other type of equation and as such not the issue here). This statement can also be expressed in the language of mathematical functions, namely as

$$y_i = f(x_i), \quad (3)$$

which can also be read as "something of the class of $y$'s, conservation in this case, is a function of something of the class of $x$'s, age in this case."

The basic difference between the current and the dynamic systems basic equation is that in the current function the domain ("input" of the function) and codomain (set of possible "outputs" of the function) are different (conservation and age), whereas in the dynamic systems function domain and codomain are the same, namely the set of possible states of the developing system (for instance conservation understanding now, conservation understanding later). Just as the dynamic systems equation $y_{t+1} = f(y_t)$ can be turned into a structure of equations of arbitrary complexity, e.g., by coupling them, the $y_i = f(x_i)$-equation too can be made as intricate as needed. It can incorporate longitudinal data, change and non-linearity, if need be. At this point, the reader might become confused: what is the point of dynamic systems if a non-dynamic systems approach can also incorporate non-linearity or present a theory of change? To clarify this point, two studies of vocabulary growth will be discussed that illuminate the core property of a dynamic systems model.

### *Example: Vocabulary growth*

The growth of the child's lexicon during the first years of life provides a good example of the application of both types of underlying model functions to the explanation of developmental processes. I shall describe two exemplary articles, the first of which represents the $y_i = f(x_i)$-family, which we have called the "standard" approach, whereas the second is an example of a dynamic systems $y_{t+1} = f(y_t)$-approach.

### *A $y_i = f(x_i)$-study of vocabulary growth*

The first, i.e., the $y_i = f(x_i)$-article, is a study by Pan, Rowe, Singer, and Snow (2005) of maternal correlates of growth in vocabulary production of one- to three-year-olds of low-income families. Their study is longitudinal and covers three measurements of spontaneous word production for each child in the sample ($N = 108$). For each child, an individual growth model was estimated, by means of a quadratic function (which means that growth is modelled as a function of an intercept, a function of age and of the square of age). Thus, you plug in an age in the equation and you get a vocabulary (this is somewhat of a caricature, but it captures the essence of the model).

In the Pan et al. (2005) study, each growth pattern, corresponding with the growth of vocabulary in an individual child, has specific properties that have been modeled as a function of maternal parameters such as the mothers' verbal and non-verbal behavior. It was found, among others, that it is primarily the diversity of the mother's language that is positively related to vocabulary growth of the child. In short, the model of vocabulary growth is like a nested model, stating first, that vocabulary is a particular function $f$ of age, and second, that this function $f$ (more precisely components of $f$, such as the slope parameter) is a particular function $g$ of maternal language input (note that $f$ and $g$ are used as so-called generic terms, meaning that the $f$ here can represent another function than an $f$ mentioned somewhere else in the article).

Note that the distinguishing feature from dynamic models is *not* that the latter are non-linear and the first are not. For instance, the growth function from the Pan et al. (2005) study is nonlinear (the reason being that $f(t_1) + f(t_2) \neq f(t_1 + t_2)$, $t$ standing for "time" or age). Individual growth models can be as non-linear as you like, but, in principle, they are always of the form $y_i = f\ (x_i)$ (for $y$ referring to age) and are thus not dynamic in the sense of the first equation, $y_{t+1} = f\ (y_t)$. In addition, it should be noted that some dynamic equations of the form $y_{t+1} = f(y_t)$ (the "dynamic family") can be rewritten as functions of the $y_i = f(x_i)$-family (for $y$ meaning age or time). For instance, the well-known logistic growth function or the generalized Richard's function of growth can be written as an iterative equation, but also as a non-iterative function where growth is simply a function of time (Banks, 2003). Thus, why do we need the iterative, so-called "dynamic" form if it can be expressed as a standard-family form anyway? The reason is that the set of dynamic equations that can be expressed as a function of time (among others) is very limited. Coupled equations of the $y_{t+1} = f(y_t)$-form (see for instance Eq. (2)) are most often not reducible to a $y_i = f(x_i)$-format. Thus, a model expressing a mutual influence over time between two or more variables, emotional expression during an interaction for instance, needs a format like Eq. (2) and cannot be replaced by a $y_i = f(x_i)$-form. Moreover, even where it is possible to do so, expressing a process of change as a $y_i = f(x_i)$-function often conceals the underlying dynamics. For instance, a dynamic model of growth can also be expressed in the form of the classical logistic growth equation, which is a representative of the $y_i = f(x_i)$-family:

$$y_i = \frac{K}{1 + \left(\frac{K}{C} - 1\right) \cdot \mathrm{e}^{-K \cdot r \cdot x_i}}, \tag{4}$$

(see Eckstein, 1999, 2000; for an application to cognitive development). The equation calculates a growth level ($y_i$) as a function of an initial level ($C$), a maximal level $K$, a growth rate $r$ and a time span $x_i$. In short, what it says it that $y_i = f(C, K, r, x_i)$ and this makes it a member of the standard family, which, in its most simplified form amounts to $y_i = f(x_i)$ ($x_i$ is the variable part in the equation, $K$, $C$, and $r$ are constants and can thus be subsumed under the $f$-part of the equation) . This equation is equivalent to saying that the next growth level, $y_{t+1}$ is a function of a growth rate $r$, a maximal value $K$ and the preceding level $y_t$ (the initial level $C$ is nothing but the first $y_t$). Stated in the latter way, it forms an example of a dynamic equation of the $y_{t+1} = f(y_t)$ form. However, unless one does the mathematics of transforming the dynamic equation (for $((t + 1) - t)$ approaching the zero limit) it is virtually impossible to see that the first equation is equivalent to the second. In short, the fact that (in reality very few) dynamic equations can be transformed into the standard $y_i = f(x_i)$-form, does not imply that the dynamic $y_{t+1} = f(y_t)$ form can be disposed of.

## *A $y_{t+1} = f(y_t)$-study of vocabulary growth*

An example of a dynamic systems, $y_{t+1} = f\ (y_t)$-type study of vocabulary growth is the study of Robinson and Mervis (1998). They describe the growth of vocabulary in one child, in relation to the growth of the use of plurals in that child. Their model, based on van Geert (1991), says that a certain level of vocabulary is a precursor for the emergence of plural forms and that the rapid growth of plurals has a temporary slowing-down effect on the growth of the vocabulary. The slowing down is probably caused by the temporary allocation of additional attention and interest in the use of a new way to make new words, namely the plural form. The model is written in the form of a set of coupled equations, one relating to vocabulary, the other to the use of plurals. The data were taken from an observational diary study of one child between the ages of 10 and 30 months and covered about 37,000 linguistic entries (see Fig. 1). It is clear that a measurement schedule of three repeated observations cannot capture the details needed to validate the underlying model. The rapid growth of plurals, for instance, could easily be mistaken for a standard process of continuous decelerating growth, represented by a second-order polynomial model.

The differences between the Pan et al. (2005) and the Robinson and Mervis (1998) studies are interesting. The first study tries to estimate the relationship between maternal language use and vocabulary growth in a representative sample of (low-income) children. To do so, the reserachers investigate an extensive sample of subjects by taking a few repeated measurements. In their model, there is no explicit dynamic function, i.e., a function that specifies the mechanism of change and thus explains how the current level of vocabulary of a particular child leads to a new (in principle higher) later level of vocabulary in the child. The dynamic function remains implicit, in that the model says that a child's vocabulary growth is stimulated by the quality of the mother's language input. The second study aims at validating a process model that is explicitly dynamic, i.e., where the next step in a coupled system of variables is a function of the preceding step. The validation requires a great number of repeated observations in very few subjects (in fact, only one). The dynamic function is explicit, but it remains unclear how it is distributed across the population. It is clear that, if there were 108 studies like this (108 is the number of subjects in the Pan et al. (2005) study), an individual growth modeling study based on a dynamic systems model of growth would be possible. That is, it would be able to obtain sample estimates of the dynamic precursor function between vocabulary and plural use, of the temporary competition relationship between plural use and vocabulary growth etc. Collecting many

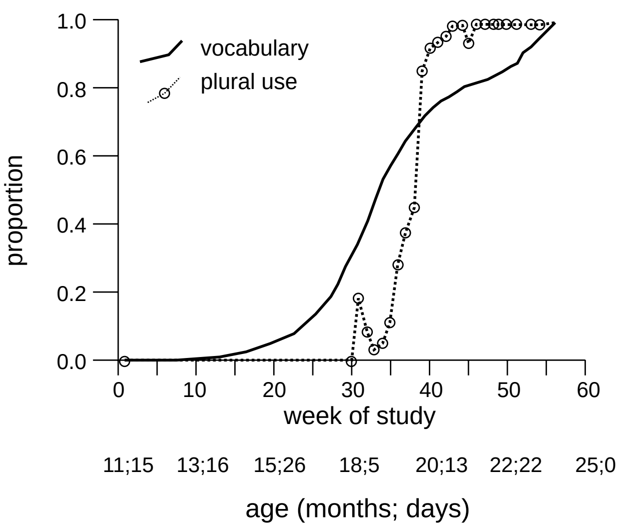


Fig. 1. Vocabulary growth and use of plurals, adapted from Robinson and Mervis (1998).

individual studies before any generalization to the broader population is made is far from an exotic idea: In the study of language development, individual studies are brought together in a comprehensive data base (Childes), which, in principle allows the researcher to combine individual growth modeling over a sufficiently extensive sample of children based on process models that describe the dynamics of the language-developmental process (MacWhinney, 2000).

### *A $y_{t+1} = f(y_t)$-reinterpretation of the $y_i = f(x_i)$-study of vocabulary growth*

To further clarify the distinction between the two types of models, we can ask ourselves whether it is possible—and worthwhile—to try to transform the Pan et al. (2005) model into a dynamic model of the $y_{t+1} = f(y_t)$-form. If this transformation succeeds, we will have two different but complementary views on the phenomenon of maternal influence on vocabulary growth. Having such alternative views may contribute to a better understanding of how these developmental views are different yet complementary.

Stating, as in the Pan et al. (2005) study, that the child's vocabulary growth is a function of the diversity of the mother's input (number of word types used) is more or less equivalent to stating that the child will probably acquire the words used by the mother and thus, the more words the mother uses, the more words the child will eventually learn. Stated in the language of growth dynamics, this is equivalent to

$$C_{t+1} = f(C_t, M_t)$$

for $f$ having the following form

$$C_{t+1} = C_t + \text{growth rate} * (1 - C_t/M_t).$$

This model says that the child's vocabulary $C$ will grow towards the vocabulary of the mother, $M$, with a certain rate of growth this particular model has been chosen because it yields an output that can be described quite well by the quadratic growth model from Pan et al. (2005); a theoretically more convincing model would be a logistic model; for a discussion of these models, see van Geert (1991).

Let us now add the assumption that—at least some—mothers are sensitive to their child's learning of vocabulary and will tend to increase the diversity of their own vocabulary as the child progresses with his or hers. That is, under this assumption, the change in the mother's vocabulary is a function of her own vocabulary and the change in the child's

$$M_{t+1} = f(M_t, \Delta C_t)$$

(the $\Delta$ stands for the amount of change).

In addition, we assume that the diversity of the mother's vocabulary will never be less than that of the child (since it is based on a communicative situation, we can assume that the "adult" words for objects, properties etc. that the child uses will also be used by the mother). These assumptions can be combined into a coupled growth equation (see website materials for details and a running model at www.vangeert.nl).

A characteristic temporal pattern resulting from this particular dynamics of interactions between mother and child vocabularies is represented in Fig. 2.

The horizontal axis represents time in weeks and thus covers a period of about one year. The vertical axis represents the child's and mother's vocabulary by means of a proportion-

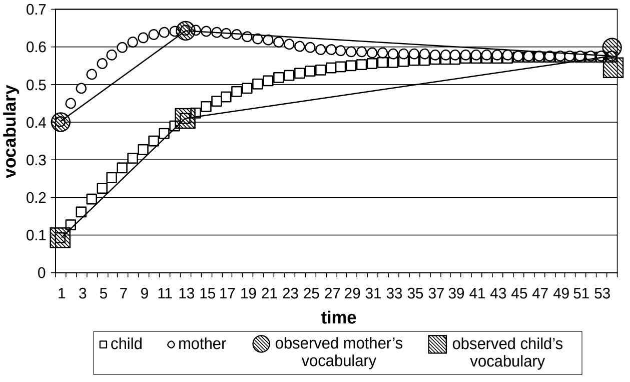


Fig. 2. A characteristic trajectory of vocabulary growth in a child, based on the dynamic model explained in the text. Circles represent the diversity (number of different words) that the mother produces during a verbal interaction with the child. This model could in principle be validated with only three repeated observations.

al figure. For simplicity, 1 represents the mother's total home vocabulary, i.e., all the words she would tend to use when talking about more or less domestic things, communicating with the members of the family, and so forth. The vocabulary that the mother uses in her communication with her young child is expressed as a proportion of her total vocabulary (e.g., 0.4 at the beginning of the modeled trajectory; these proportions are chosen only for reasons of demonstration and do not refer to any empirically verified figure). The child's vocabulary can be expressed on the basis of a similar, proportional measure. For simplicity, the model assumes a lexical "status quo," i.e., a relatively fixed final vocabulary used in the communication with the child, about child- and domestic matters. In reality, this vocabulary will increase when the child goes to school or when the child starts to draw from other linguistic sources than the mother. Thus, the validity of the model is limited in terms of the time span it covers.

Under this model, the mother's input shows a transient period of extra enriched language, that settles back to an equilibrium level as the child reaches his or hers (temporary) asymptote of vocabulary (note that the vocabulary in the original Pan et al. (2005) study was measured as the number of different words over 10 min of mother–child interaction; this number is almost certain to reach a limit as the child increases his vocabulary). The minimum number of observations to validate this model is three, provided the second observation captures the predicted temporary vocabulary overshoot of the mother. In practice, however, vocabulary measures fluctuate considerably, and thus we would need considerably more observations to reliably validate the current growth model (van Geert & van Dijk, 2002; van Geert & van Dijk, 2003; van Dijk & van Geert, 2005; van Dijk, de Goede, Ruhland, & van Geert, 2001).

Instead of plotting the trajectory of vocabulary growth in a particular child over time, it is also possible to plot the outcome (the child's vocabulary level after a sufficiently long growth period) as a function of the parameter values that have an effect on that outcome. Fig. 3 shows the effect of various growth rates (ranging from 0 to 0.1; horizontal axis) on final vocabulary level (vertical axis) for four levels of maternal sensitivity (0.5, 1, 1.5, and 2, respectively). For instance, with a growth rate of 0.05 (see the position on the horizontal axis) and maternal sensitivity of 2 (line consisting of circular markers), the final level of vocabulary in the child is about 0.8.

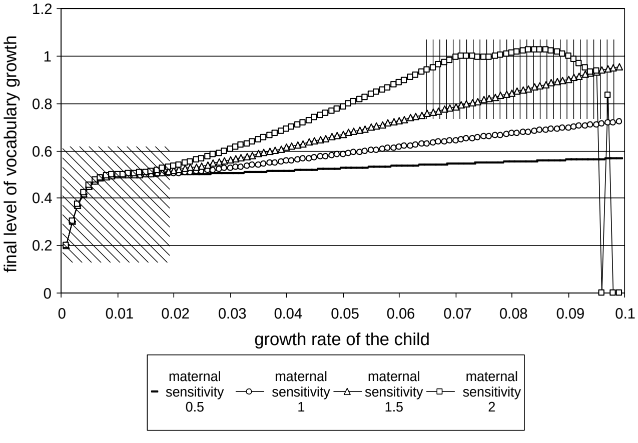


Fig. 3. In the dynamic model, the final level of vocabulary growth (vertical axis) in the child is a non-linear function of the rate of growth (horizontal axis). The sensitivity of the mother to the child's growth of the vocabulary interacts with the child's growth rate (four different sets of outcomes corresponding with four levels of sensitivity). For low growth rates, the effect of maternal sensitivity is hardly discernible between the sensitivity conditions, while the effect on the final state of vocabulary growth is highly non-linear (diagonally hatched region). For high growth rates and high sensitivity of the mother, the effect on the child's final level of vocabulary becomes instable (vertically hatched region).

The figure shows that the final vocabulary level is a non-linear function of the child's rate of growth. High sensitivity in combination with high growth rates lead to instable states (sudden drops of final vocabulary level at the right of Fig. 2). Such instabilities are theoretical phenomena, resulting from the fact that the current models are deterministic. Empirically, such instabilities probably fluctuate less wildly than those of the deterministic model.

The model presented in this section is speculative. It serves as an example of how a $y_i = f(x_i)$-type model can be linked to (and backed up by) a $y_{t+1} = f(y_t)$-type model. The $y_{t+1} = f(y_t)$-type model can generate new, testable hypotheses that the first type of model did not predict and thus increases our understanding of the mechanism and course of vocabulary growth.

Finally, the current dynamic model takes the process of vocabulary growth, the learning of new words, for granted and does not provide an explanation for the learning itself, i.e., it does not provide a dynamic model of how single words are actually learned. Connectionist and associationist models might provide a solution to explaining the learning process dynamically (Colunga & Smith, 2005; Jones & Smith, 2005).

## *What is it that a dynamic model applies to?*

### *Does it apply to something real, or to something just metaphorical?*

Growth models of the type discussed above are a little suspect, at least according to some critics whose competence in dynamic systems cannot be doubted (Kelso, 1995). The question is simple: what is it that the dynamic growth model applies to? The model applies to vocabulary, for instance, but vocabulary is an abstraction. It is not "vocabulary" that generates the words that a child produces and that we use to calculate the child's vocabulary. In fact, we still have only little knowledge of what exactly is responsible for the production of words. We know for instance that particular brain regions play an important role but the knowledge is still limited (Hickok, 2001; Poeppel & Hickok, 2004). It is often difficult to determine where one phenomenon (e.g., vocabulary) ends and another (e.g., grammar) begins. Compare this with fields in which studies of growth dynamics abound, such as ecology. Damgaard and collaborators, for instance, modeled the growth and competition of Chenopodium Album, a relatively common plant better known as lamb's quarters or pigweed (Damgaard, Weiner, & Nagashima, 2002). A botanist can clearly distinguish lamb's quarters from some other plant growing in the same meadow. It is clearly defined as a physical thing and its physical growth and reproduction is well understood. It competes with other plants for nutrients, physical space and sunlight, each of which are physical quantities that can be measured precisely. In comparison with this, "vocabulary," "plurals" and their eventual competition for resources are somewhat ghostly apparitions which have no clear physical counterparts. Applying the notion of "growth" to these phenomena cannot be but metaphorical, and therefore any dynamic model of their growth and competitive interaction must be metaphorical too. Can a model that pretends to be scientific be metaphorical?

*The "Bloomington approach"*

In developmental psychology, proponents of dynamic systems have taken two different stances with regard to the problem of the eventual metaphorical nature of the phenomena their models address. The "Bloomington view" (Thelen, Smith, and others) sees this problem as a major problem of psychology, namely that psychology overall tends to invoke ghostly things to explain behavior. Traditional psychology, Piaget among others, explains the baby's A-not-B error by the absence of a fully developed object concept and actions such as the finding of an object hidden behind a screen by the presence of an object concept (Smith, Thelen, Titzer, & McLin, 1999). According to the Bloomington view, invoking the notion of "concept" to explain a particular behavior or action related to that concept, is a categorical error, as if one explains the color of the red traffic light by the workings of its inherent redness. It is this categorical mistake that Thelen and others seek to repair by explaining phenomena such as the A-not-B-error by a theory of situated action, in which an embodied subject (not an epistemic subject, such as in Piaget's case) acts with the help of—and under the constraints of—a physical world that includes the external environment and the physical properties of the body including the brain (Thelen & Smith, 1994, 1998). Thus, with regard to the A-not-B-error, the baby's representation of spatial locations consists of the baby's brain holding a field of activation weights, the properties of which link to the structure of space (Schutte & Spencer, 2002; Schutte, Spencer, & Schöner, 2003; Thelen, Schöner, Scheier, & Smith, 2001). The mathematical properties of that field can be defined rigorously and allow for a dynamic systems model that is no longer just metaphorical (Schöner, 2005). The dynamic field theory that describes the dynamics of this field thus bridges the "representational gap" that exists in current dynamic systems models (Spencer & Schöner, 2003). This "representational gap" refers to the fact that Bloomington dynamic systems model have no use for concepts and representations as mental entities that act as mental causes of behavior (Colunga & Smith, 2005).

In short, by linking human action and cognition explicitly to physical time and space and to brain-based activation fields, the metaphor is taken out of the theory.

*The "Groningen approach"*

The "Groningen brand" (if we may take the liberty) takes the issue considerably more lightly and philosophically in-the-European-manner (if that makes any sense). A couple of years ago, Esther Thelen asked one of the Groningen PhD students what the topic of her dissertation research was. The student replied "Theory-of-Mind" and Esther replied, "Well, we do not believe it exists" (knowing Esther's commitment to young researchers, the gist of that remark was no doubt didactic and not ontological....). The Groningen point of view is that complex human behaviors, which are meaning-laden and for a great deal invoke the use of (verbal and non-verbal) symbols, should not be described as metaphorical entities, which thus remain vague, intangible and arbitrary, but must be described by means of Haken's notion of order parameters (Haken, Kelso, Fuchs, & Pandya, 1990; Haken, 1999; Kriz, 2001; Latané, Nowak, & Liu, 1994).

Let us try to explain this point by focusing on some episode of interaction, for instance a conversation between two persons. This interaction event is characterized by a nearly infinite number of properties, each of which an observer could eventually focus on (such as the persons' positions in space, their motion, their temperature, etc.). There are many different ways in which the information gathered could be described (e.g., movement of the facial muscles described either as physical muscular changes or as an emotion) and there are a myriad of different ways to anticipate its future state. Of these nearly infinite many ways of capturing the event of an interaction between persons, human observers will usually spontaneously zoom in onto one particular pattern, which invokes highly specific information about the event (e.g., about what the participants say, about the motion of their facial muscles, etc.) and highly specific symbolic descriptions invoking words such as "want," "disappointed," etc. This particular pattern is the "order parameter" that describes the event of social interaction perceived by an ordinary human observer and the mechanism that explains its emergence in the case of this particular event is called the observer's "theory-of-mind." We call it the observer's "theory-of-mind" for little other reason than that almost everybody in the field would tend to call it so. In normally developing persons from about four to five years old, the patterning of the information obtained from observing an interaction between persons occurs immediately and spontaneously, whereas children with autism disorders have difficulties with such spontaneous patterning, i.e., with imposing this particular order parameter onto the information they obtain (Blijd-Hoogewys & van Geert, 2005; Serra, Loth, van Geert, Hurkens, & Minderaa, 2002; Serra, Minderaa, van Geert, & Jackson, 1995). Calling the mechanism that is responsible for this spontaneous patterning of information the person's "theory-of-mind" does, in our framework, not imply any particular properties that mechanism should have. That is, it does not imply that it should be a "theory" or that it should be some mental structure.

*The notion of "generator" and an example from Theory-of-Mind*

A generally applicable term for all sorts of mechanisms that are responsible for the production of some sort of psychological phenomenon, action, knowledge, etc. is the term "generator." We have only very little knowledge about the properties of the theory-of-mind generator, but it is likely that it involves both internal and external components, that it involves context-specific couplings between internal and external properties, i.e., properties of the person and of the environment. In short, the nature of the theory-of-mind generator is likely to resemble the type of generator that causally explains why some infants make the A-not-B-error, in that it involves loops of internal "effectivities" (i.e., abilities, possibilities of action) and external "affordances" (Clark, 1997; Clark & Chalmers, 1998; Shaw, Turvey, & Mace, 1981). To understand the way in which this and comparable generators work, a detailed short-term observation of the relevant events is necessary, for instance, a detailed study of the eye movements, imagery, covert and overt verbal and non-verbal symbols generated during the very short time span that a "theory-of-mind"-interpretation comes about. It is likely that such study lies beyond what psychology can accomplish at the moment. With simpler generators, such as the coming about of an A-not-B-error in a baby in a particular context, considerably better insights have been achieved, as the aforementioned studies testify of.

To study the development of a generator (e.g., of vocabulary, of social understanding, etc.) one needs a way to capture the generator's change. We have referred to this requirement earlier, in the section introducing the basic equation of dynamic systems, in a somewhat different form, namely how to distinguish the different "states" that a dynamic system entails. In the case of a phenomenon such as vocabulary, change may be measured by counting the number of new words in spontaneous speech. With a phenomenon such as theory-of-mind, change is more difficult to track. This problem relating to developmental rulers and metric spaces will be discussed in the next section.

A second problem, to be discussed in the section after the next one, relates to the fact that, apart from trying to understand a particular generator at the short-term time scale of concrete action, it is important, especially from a developmental point of view, to try to understand its long-term time scale properties. For instance, over the course of months or years we see changes in the long-term properties of the object concept, vocabulary and grammar, theory-of-mind and so forth. These changes must be the consequence of the way the short-term patterns operate, e.g., of the way infants manipulate objects, of the way children are involved in social actions and so forth.

### *So, in the end, what is it that a dynamic model applies to?*

In summary, the question whether dynamic systems models apply to something real or something just metaphorical can be answered the Bloomington way and the Groningen way. According to Thelen and colleagues, it must apply to real action on physical objects in real time and space (assuming that "objects" can have a broad meaning, including words spoken, physical symbols and so forth). According to the "Groningen" approach, followed by the present authors, the incredible complexity of the brain and the corresponding complexity of the world are characterized by complex patterns, i.e., order parameters, that emerge out of the physically based interactions between organisms and the environment. This is the sort of complex order and pattern that psychologists and laypersons alike are used to refer to by means of "psychological" terms such as "concept," "theory-of-mind" and so forth. Calling them by these names does not automatically involve an ontological claim, as if concepts would be the little engines inside a person's head that generate particular behavior. These order parameters or patterns can be assigned certain macroscopic properties, such as a certain quantitative level (in the sense that a child's theory-of-mind can be assigned a certain level of development, in comparison to other children or in comparison to an earlier age; compare this with the physical notion of "temperature," which is a macroscopic property of an object and in fact amounts to the intensity of the motion of the molecules that constitute the object). These order parameters or patterns entertain specific relationships with each other, which can in principle be formalized and quantified, and thus can be the subject of dynamic models. The focus on the properties of and relations between macroscopic patterns or order parameters is perhaps the most characteristic feature of the Groningen approach, as it has been called in this article. These macroscopic patterns are similar to what in the standard approach is called "psychological variables." In this sense, the Groningen approach stands closer to the standard view than the Bloomington approach does, and thus shares questions with the standard view such as "how does theory-of-mind develop" or "how are syntactic and lexical development related?." However, Groningen and Bloomington share the belief that the macroscopic level of description has its complement in a microscopic level, where the generator of the macroscopic level amounts to physically embodied action. The standard psychological view, which Thelen and colleagues have so strongly criticized, is that the macroscopic level of behavior corresponds with a generator level that is equally macroscopic, but of a different, namely mental or psychological nature.

*Metric spaces and the need for developmental rulers*

In the example about Piaget and cognitive development we have seen that a dynamic model—Piaget's—can be formulated over a space of change that consists of a number of separate states (developmental stages). However, in the preceding summary section we have hinted at macroscopic properties of behavior, such as Theory-of-Mind or Vocabulary, that can be assigned quantitative measures. To describe such quantities, you need a space of change that consists of dimensions that take the form of a number line. Put differently, you need a so-called metric space. The metric space consists of the time dimension and at least one other dimension measuring the variable at issue (vocabulary, theory-of-mind, cognitive complexity,...). To specify the position of a variable in this space one needs a clock (to specify the time, e.g., age) and a ruler, to measure the size of the variable. Developmental rulers, as Kurt Fischer calls them ( Fischer & Rose, 1994; Fischer & Dawson, 2002) are of central importance to any study of development that tries to capture the developmental dynamics of a particular variable or phenomenon. An example of such a developmental ruler is a test, for instance a theory-of-mind test that is able to reliably capture the change in theory-of-mind understanding over the years. However, a single task or test will not provide an adequate developmental ruler, because it is usually too limited in the number of contexts and behaviors it addresses, and thus, a collection of such tests or tasks is required (Fischer, 1987, 2005; Fischer & Rose, 1994; Fischer & Bidell, 2005; Rose & Fischer, 1998). The dimensions or variables constitute the phase or state space within which the phenomenon at issue varies and in which its growth or development can be specified in terms of its position on the developmental ruler(s).

If the construction of such rulers is not feasible, change can be described in terms of categorical spaces, as was the case with the example of Piaget. Another example is social and emotional interaction between parents and children. Such interaction can be described in a two-dimensional space. One dimension specifies the behavioral state of one person and the other dimension the behavioral state of the other person. In our own study of social interaction in young children that will be described later, we distinguished two behavioral states, namely " other-directed action, i.e., any action addressing the other person" versus "solitary action, i.e., action focused on one self." These two categories apply to the participants in a dyadic interaction and thus constitute a space with four possible states (e.g., one state is that person 1 addresses person 2 and person 2 continues his solitary action). With more behavioral categories one obtains a grid. The changes, for instance in dyadic interaction, over this grid can be statistically analyzed. A good example of a method that does such an analysis and comes with associated software is the state space grid method (Granic, Hollenstein, Dishion, & Patterson, 2003; Granic & Lamey, 2002; Hollenstein, Granic, Stoolmiller, & Snyder, 2004; Lewis, Zimmerman, Hollenstein, & Lamey, 2004). Another method uses so-called Karnaugh maps, which are based on binary dimensions, any number of which can be compressed into a two dimensional space (Dumas, Lemay, & Dauwalder, 2001). Once such a space is constructed, any mathematical or otherwise formal expression can be used to specify the change of the variable across this space, i.e., the variable's dynamics.

In the behavioral sciences, the relation between the particular rulers employed on the one hand and the underlying assumed dimensions or variables has always been problematical (a typical question here would be: what is the relation between a child's score on a Theory-of-Mind test and the child's actual Theory-of-Mind). This is especially true for the study of development. Measured variables, such as the average number of different words used in a communicative interaction of 10 min, fluctuate from day to day. To what extent is this fluctuation part of the developmental change and to what extent is it measurement error? According to dynamic systems theorists, fluctuation is information, not measurement error (Thelen & Smith, 1994; van Geert & van Dijk, 2002). Moreover, if one is observing something that is still in the making, i.e., that is still developing, it is sometimes hard to decide if a particular observation is indeed an instance of the phenomenon at issue or not. For instance, in adult language, a particular word—for instance "on"—belongs to a particular linguistic category, for instance a preposition, or not. In early language development, however, linguistic categories such as verbs and prepositions are still being formed. In early child language, a word such as "on" may have some properties of a preposition and some of a verb. This ambiguity of observations is typical of a dynamic developmental process, for instance of the transformation of a proto-word into a genuine preposition. It is different from the problem of uncertainty: uncertainty comes from lack of information, whereas ambiguity, vagueness or fuzziness is a property of the observed phenomenon itself. In a dynamic systems account of development, variability, fluctuation, ambiguity and vagueness of observed phenomena must be accounted for explicitly. These properties provide information about the underlying dynamics of the developmental process (van Dijk & van Geert, 2005, 2002; van Geert & Van Dijk, 2003).

### *Time scales and an example from social interaction and its development*

As announced in the preceding section, a thorough understanding of the long-term dynamics of a developmental process requires its coupling with the short-term dynamics of the phenomenon at issue. In our own research, we have tried to combine the long-term dynamics of social development in children with a model of the short-term dynamics of social interaction between children, dyadic interaction in particular.

### *Dyadic interaction and social status in children*

The question of the study is as follows: Is there a different pattern of dyadic interaction in six- and seven-year-old children of different sociometric statuses and how can these differences in patterns be explained (Steenbeek & van Geert, 2005a, 2005b, 2005c; Steenbeek & Van Geert, 2002). There exists an extensive literature showing that in children there exists an association between adequate and emotionally positive interaction patterns with high social status on the one hand, and less or inadequate and emotionally negative interaction patterns with rejected status on the other hand. Moreover, children who lack adequate social skills run a greater risk of receiving a rejected sociometric status in the class (see Steenbeek & van Geert, 2005a for an overview of the literature). It has also been shown that especially when this status remains relatively stable; the child runs the risk of encountering problems later on in life (Cillessen, Van IJzendoorn, Van Lieshout, & Hartup, 1992; DeRosier, Kupersmidt, & Patterson, 1994; Kupersmidt & Coie, 1990). In short, there appears to exist an interaction between social status, social skills and social interaction, both in the long-term (social development) and the short term (social interaction). How do these two time scales relate to one another?

### *The dynamics of short-term dyadic interaction*

To begin with, it is interesting to see that the notion of dynamics has played an important role in social psychology, at least beginning with the work of Kurt Lewin who related dynamics with social interaction and personality (Lewin, Adams, & Zener, 1935). The focus on the dynamics went along with an emphasis on individuals and their interaction with others, which is the level at which the dynamics applies (Herbst, 1953, 1954, 1957). Various authors have suggested dynamic approaches of dyadic interaction (Dishion, Bullock, & Granic, 2002a, 2002b; Dishion & Dodge, 2005; Dishion, Poulin, & Burraston, 2001a, Dishion, Poulin, & Burraston, 2001b; Felmlee & Greenberg, 1999; Gottman, Guralnick, Wilson, Swanson, & Murray, 1997; Gottman, Murray, Swanson, Tyson, & Swanson, 2002) and dynamic approaches of social interaction in larger groups (Dumas et al., 2001; Nowak, Vallacher, & Tesser, 2000; Vallacher & Nowak, 1997; Nowak & Vallacher, 1998; Vallacher & Nowak, 1994; Vallacher, Nowak, & Collins, 1995).

In our own model of dyadic interaction between children, we started with what the literature has to say about the fundamental properties of the dynamics of social or dyadic interaction (see Steenbeek & van Geert, 2005b). These fundamental properties are the following.

### *Goal-directedness and appraisals*

The first property is that action and social interaction, for that matter, is goal-oriented (Austin & Vancouver, 1996). At first sight, goals or intentions seem like very explicit internal representations, mental phenomena that anticipate and govern action. This mentalist view on goals is in discordance with the dynamic systems view on action as advocated by Thelen and Smith, among others. According to the literature, however, the mentalist view is also discordant with the facts: goals and intentions are largely unconscious and emerge under the control of the context (Bargh & Chartrand, 1999; Bargh & Ferguson, 2000; Chartrand & Bargh, 1999). Various authors, working in a dynamic systems framework, have argued that a non-representationalist dynamic systems treatment of intentions and goals is very well possible (Gibbs & Van Orden, 2003; Juarrero, 2002; Kappas, 2002; Shaw, 2001).

A second general principle of action and social interaction is that goals represent interests and that interests, or concerns, are constantly evaluated in terms of various emotions. This is the central tenet of appraisal theory (Frijda, 1986; Roseman & Evdokas, 2004; Scherer, 1999). Working from a general, biological perspective, Cabanac (2002, 1992) has argued that all these emotions have one underlying "currency" in common, namely pleasure. Pleasure forms a general, spontaneous evaluative dimension of all action and is closely related to emotional expression (Cabanac, 2002; Johnston, 2003; Panksepp, 2000). By linking appraisals to this underlying pleasure dimension, appraisal models can easily be connected with learning-theoretical models, especially those referring to the Matching Law (Steenbeek & van Geert, 2005b). By doing so, the dynamics of interest- or concern-related actions can be described as a process of optimizing the value of the pleasure dimension in a particular context and on a particular time scale.

*Social interaction as a goal of social interaction*

These two general properties can be applied to social interaction in a simple, self-referring way: an important, implicit goal of social interaction is to socially interact with another person. With some persons, that interaction is more pleasurable than with others. Moreover, preceding a potential interaction, some persons are preferred over others, for instance because they are important or powerful in the group. This preference is another way of expressing the intentionality or goal-directedness of social interaction, which is, as previously stated, usually not conscious or explicit. However, not all action in social interaction is social interaction. That is, a participant may address another one verbally and or non-verbally, for instance by means of an emotional expression. The other person need not necessarily react to that and may go on doing his own business for the time being, without really responding to the first person. Real interaction requires a level of mutual adjustment of the actions that can be referred to by terms such as mutuality, reciprocity or coherence. In short, social interaction as a goal can only be satisfied if there is sufficient reciprocity in the social actions of the participants (Fogel, 1993; Steenbeek & van Geert, 2005a, 2005b, 2005c).

*Contagiousness, imitation, and emulation*

The last component of action that is particularly relevant with regard to the reciprocity or involvement aspect concerns the fact that action is to a great extent a socially contagious phenomenon. People tend to automatically adopt the behavior of other persons, partly because the other person is the context of their own action, partly because this is a natural tendency of action in social beings. The contagiousness of behavior has been well documented in social psychology, for instance in social learning theory (Bandura & MacDonald, 1963; Bandura, 1977) and in the literature on social, emotional and behavioral contagion (Levy & Nail, 1993; Nail, MacDonald, & Levy, 2000; Neumann & Strack, 2000; Wheeler, 1966). The tendency to imitate others is a biological property of our species and is neurologically specified by the presence of so-called "mirror neurons," which are not just blind copying machines but actually help people (or individuals of the primate family in general) understand the intentions of others (Fogassi et al., 2005; Gallese, Keysers, & Rizzolatti, 2004; Preston & de Waal, 2002; Rizzolatti & Craighero, 2004). This understanding is not an explicit mental representation, but a consistent, reciprocal responding to the actions of another person, where the other "emulates" the goal of the first. This emulation occurs by following or extending the dynamics of the other person's actions in a way that exceeds the purely spatio-temporal, physical aspects of the actions involved (Call & Carpenter, 2002; Carpenter & Call, 2002; Carpenter, Call, & Tomasello, 2002; Carpenter, Call, & Tomasello, 2005; Thompson & Russell, 2004).

### *Social status and social interaction in children*

Finally, how does social status, which was the starting point of the particular research project discussed here, relate to this conceptual model of the dynamics of social interaction? The answer lies in the assumed relationship between the social status that a group assigns to a particular child, i.e., the child's social power in the group, and the child's presumed social competence. The social psychological literature has long since emphasized the fact that people tend to accommodate to popular, i.e., high power individuals in their group, that they prefer to interact with more than with less popular or powerful subjects and so forth (Copeland, 1994; French & Raven, 1959; Raven, 1992; Snyder & Kiviniemi, 2001). Thus, it is expected that children will have a higher preference for interacting with partners of higher social power, i.e., more popular children. It is likely that this preference is expressed in the form of pleasure or joy during an interaction. This preference is not some anticipatory state of mind or an explicit goal representation. It is a spontaneous tendency to act towards an optimization of appraisal or pleasure, which has a higher equilibrium level with popular play partners than with others.

In addition, social status is associated with social competence, which can have both an explicit and an implicit aspect. The first refers to a person's ability to achieve his or her social goals with minimum effort ("effectiveness"). In general this ability is likely to be higher in children of higher social status (Hazen, Black, & Fleming-Johnson, 1984; Rose-Krasnor, 1997; Rubin, Bukowski, & Parker, 1998; Simeo-Munson, 2000). The more implicit aspect refers to the fact that children with higher social status have a stronger "contagiousness" level, i.e., that children tend to imitate popular children more easily than less popular children.

### *A dynamic systems model of dyadic interaction in children: Model and results*

All these elements served to formulate two dynamic systems models of social, more precisely dyadic, interaction. One model consisted of two coupled difference equations specifying the proportion "involvement"-behavior over the short time span of a dyadic interaction (Steenbeek & van Geert, 2005b). The other model was more extensive and consisted of an agent model, incorporating the same theoretical principles, but yielding two different "outputs," namely emotional expressions on the one hand and actions that are either other-oriented or self-oriented on the other hand (Steenbeek & van Geert, 2005a, 2005c). The models were validated in the following way.

### *Empirical data*

Twenty-four dyads of grade 1 pupils with mean age of 6.5 years participated. Each dyad consisted of two same-sex children. Three types of dyads were formed, one consisting of a child with rejected status and an average status play partner (the "rejected" dyad), two average status children (the "average dyad") and finally a popular child with an average status play partner (the "popular" dyad). The dyads were videotaped during a 10-min free play session with a standard set of toys. The 24 dyads were videotaped three times, with intervals of approximately one and a half month. The second and third videotaped interactions were chosen for coding. Due to practical limitations, 7 tapes were not coded, resulting in a total of 41 coded sessions.

Two variables were coded by means of event sampling (with a precision of 1/10th second): *emotional expressions* and *instrumental actions* of each child separately. The variable *emotional expression* was coded on a scale ranging from very negative (−4) to very positive (+5). The variable *action* was coded with the help of three overt variables: verbal turn, non-verbal turn, and focus. On the basis of these partial variables, a child is coded as displaying *other-directed action* (Playing Together) or self-directed action (Playing Alone, i.e., solitary action). The duration of these actions is estimated with a precision of 1/10th second. Other-directed action also entails *attempts* towards involving the other child in the interaction. If at time $t$, both the child and the play partner show mutually responsive *other-directed actions*, the behavior is coded as "*dyadic coherence*," which is the only action variable on the dyad level.

## *Results from the coupled-difference-equation model*

The model basically says that the interaction is based on three components that are evaluated and updated at each time step. The first component relates to the fact that pleasure will be maximal for a specific proportion of "real," i.e., mutual and coherent interaction in the interaction situation (i.e., other-directed actions that are coherently responded to by the other person). By means of a so-called "hill-climbing" process, the frequency of other-directed action will be varied until an optimal level of real interaction is achieved and thus, an optimal appraisal level is achieved (details are discussed in Steenbeek & van Geert, 2005b). The second component is the "contagiousness" component and involves an adaptation of the child's level of interaction to that of the play partner. The third component is an "influence" component and contains attempts to adapt the play partner's level of interaction (involvement) to the child's preferred level. The influence component is moderated by a social-competence parameter which varies with social status. The first and second component are sensitive to the play partner's social status (e.g., with a popular play partner, a child will aim at more intensive and longer interaction, all other things being equal).

These model components are implemented in the form of the following coupled equations

$$\Delta I/\Delta t = a(P_I - I_t) + b(P_I - Y_t) + e(Y_t - I_t),$$

$$\Delta Y/\Delta t = c(P_Y - Y_t) + d(P_Y - I_t) + f(I_t - Y_t).$$

"$I$" stands for the level of other-directed action by the child, "$Y$" is the level of other-directed action by the play partner. "$P$" refers to the preferred level of social interaction in the child and the play partner and depends, among others, on the social status of the child with whom one is playing; a to $f$ are parameters contributing to the rate of change in $I$ or $Y$. $\Delta I/\Delta t$ means "the change in $I$ over a time interval $\Delta t$." This equation is a typical dynamic systems model, although, superficially, its form is different from the standard form used in this article. However, set $\Delta t$ equal to 1 and the equation becomes a version of the standard dynamic equation $I_{t+1} = f(I_t)$, with $f(I_t)$ equal to $I_t + \Delta I_t$.

Given specific values of the parameters, the model produces a specific equilibrium level for the frequency of the reciprocal social interaction in a particular dyadic situation.[2] The model was primarily used to generate a distribution of average levels of mutual social engagement in a variety of dyadic interactions between children of different social statuses and by doing so predict the distribution found in the 41 coded play sessions, divided over the three, status-related types of dyads.

Validation of the model's predictions for the three types of dyads was carried out in accordance with the following procedure. First, a parameter space was defined that specifies the hypothesized values of the parameters governing the interaction dynamics for each of the dyad types. Second, the dynamic model was run for a set of parameter values that covers this parameter space (a total of 10,000 runs). Third, on the basis of these model runs, distributions of values for each of the dyad types were calculated and then compared with the observed distributions.

The results are roughly as follows. The model provided a statistically significant fit of the overall pattern of differences between rejected and popular dyads. On the level of the 10 separate variables that were tested, eight variables were consistent with the prediction in terms of the direction of difference between rejected and popular dyads and of the effect size. These variables were the following (''child'' refers to either the rejected of popular child in the dyads, ''play partner'' refers to the average-status child in the dyads): the percentage other-directed actions of the child and of the play partner; the percentage reciprocated other-directed actions (i.e., those that were responded to by an other-directed action of the other child and thus refer to mutual action or interaction); amount of positive expressions of child and play partner and amount of mutual positive expressions (when a positive expression of one child is answered by a positive expression of the other).

Of these 10 variables, four referred to the proportion of mutual action or expression over the total other-directed action or expression. For instance, if the child shows other-directed action during 70% of the play time, and this other-directed action is reciprocated by the play partner in 50% of the cases, the child's *proportion* of shared action is 50% (but the *percentage* of shared action across the play time is 70% times 50%, i.e., 35%). The *proportion* of shared action is an indirect measure of the effectiveness of the child's social actions: a socially effective child succeeds in making the other child respond coherently more often than a less socially effective child. Two of these variables (proportion shared directedness of the play partner and proportion shared positive emotions of the child) were consistent with the data, two others were not (proportion of shared directedness of the child and proportion shared positive expression of the play partner; for further explanation, see Steenbeek & van Geert, 2005b). In summary, it seems as if the model falls short in its explanation of variables that depend on social competence.

[2] A glance at Fig. 5 shows that the real interaction process shows a cyclical pattern, i.e., oscillations in the frequency of the interaction. The current model does not show these oscillations, but it will do if the model equations are slightly altered: instead of taking the state of variables $I$ and $Y$ at the preceding, single moment $t$ as the input of the next iteration, one can also take the average of preceding states over some past interval, 20 seconds for instance, as input of the next iteration. This so-called integration over a past episode results in an oscillating pattern, with an average value equal to the equilibrium value produced by the current model.

## Results from the agent model

The second, i.e., agent model consists of a considerably more extensive set of coupled dynamic equations than the first model. The equations are referring to model components such as preference, emotional evaluation, coupling with emotional expression, effect of the other child's action, effect of emotional expression and so forth. The relations between the components are represented graphically in Fig. 4. The arrows refer to simple quantitative relationships. White arrows refer to relationships within a single time step, black arrows to relationships between time steps. For instance, the difference between preferred and realized level of real play interaction at time $t$ determines the strength of the drive to produce an other-directed play action, which is nothing but a probability that such action will be carried out at time $t$. Another example relates to the arrows between emotional expression and preference for either a play-together or play-alone action. For instance, if a play-together action at time $t$ is accompanied by a positive expression, the preference for play-together actions is updated at time $t + 1$, namely by slightly increasing this preference.

Just as the first model, the agent model can predict distributions of average interaction levels and average emotional expression levels for samples of dyads consisting of children with different social statuses. An advantage of the agent model is that it produces more and also more detailed output variables than the coupled equation model (e.g., it shows changes in the level of other-directed action over the course of a simulated play situation).

To validate the model, we proceeded as follows.

First, we checked whether the model yielded total values, averages of these totals and distributions of these totals that were consistent with those found in the data (for instance,

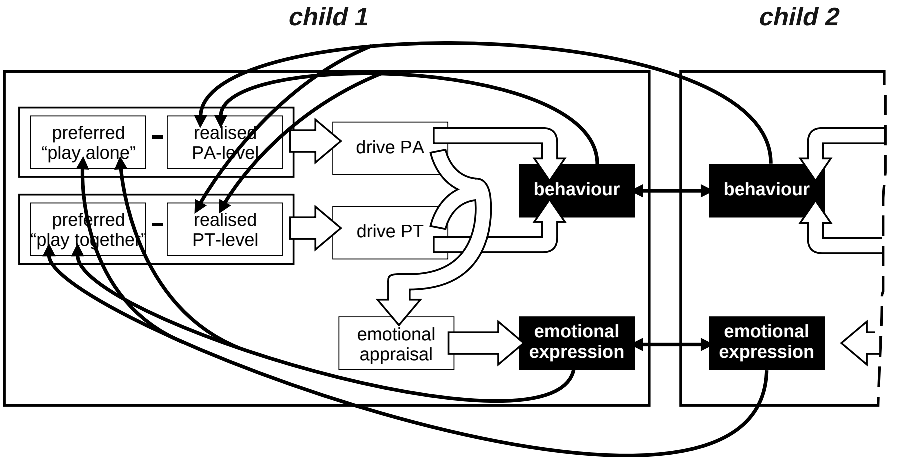


Fig. 4. A graphical representation of the agent model of dyadic interaction. The "child 2" box has reduced detail, for simplicity, but is a copy of the "child 1" box. Context- and time-specific preferred levels of playing alone with the toys or playing together with the other child are evaluated against the actual or "realized" levels. This evaluation leads to a drive and an appraisal, which relate to a particular behavior and a particular emotional expression generated at time $t$. The black arrows refer to quantitative updates of the variables from time step to time step (from $t$ to $t+1$). They refer, for instance, to the effect of a positive expression accompanying a particular action mode (e.g., playing together at time t) on the preference for this action mode (on time $t+1$).

the total amount of other directed action in the child and play partner in a particular rejected dyad; the average of these totals for the 13 rejected dyads, and the distribution, i.e., histograms, of these totals for the 13 rejected dyads). Goodness-of-fit was determined by means of easily interpretable distance measures, such as the sum of absolute differences between the predicted and the observed averages of all variables. Statistical testing was done by means of random permutation tests, which make it possible to calculate $p$ values for any desirable type of goodness-of-fit measure (for an explanation, see Steenbeek & van Geert, 2005b, 2005c). Examples of variables that provided a good fit ($p < 0.05$) are: the percentage other-directed action and positive expressions in the popular or rejected child in the dyad, and the percentage mutual, i.e., ''real'' interaction in the dyads. An example of a variable for which the model did not predict the differences between status groups found in the data, is the proportion shared other-directed action in the play partner (i.e., how much of the play partner's other-directed actions are directly responded to by the child). As stated earlier, this variable indirectly refers to the effectiveness of social action. This is the type of variable that also the coupled equations model did not fit very well, which suggests that both models are still not sensitive enough to the social competence aspect that governs the effectiveness of a child's other-directed actions.

Second, we checked whether the agent model generated *trajectories* of other-directed action and of emotional expression that are similar to the observed ones. The question is whether the observed trajectories are similar to those found in the simulated ones. Fig. 5 provides an arbitrarily chosen example of an observed and of a simulated trajectory (the first dyad of the set of 13 rejected dyads, and the first output of a simulation run of the model).

The trajectories have been smoothed on the basis of the raw data by means of a Savitzky-Golay smoothing procedure, a standard procedure for flexible smoothing of time series (Simonoff, 1996). It is clear that the simulated and observed trajectories are far from replicas of one another. However, interaction is an adaptive and variable process, and literal similarity between a model and an observation cannot be the issue here. What is at stake is whether the characteristic qualitative properties of the observed trajectories are similar to those found in the simulated ones.

A first, major qualitative property of real interaction is its coherence or mutuality (Fogel, 1993). On a micro-time scale, mutuality involves a successive process (for instance, I hand you over a Playmobil figure, you take it). On the larger time scale of the smoothed frequencies of other-directed action as shown in Fig. 5, mutuality is expressed in the form of covariation. For instance, if one child increases its other-directed action (e.g., makes more attempts to involve the other child in a particular play action), mutuality implies that the other child also shows an increase (the other child responds to the interaction initiatives of the first and thus enhances the first child's other-directed action). Likewise, mutuality requires that if the intensity of other-directed action in one child diminishes, it should also diminish in the other child. This type of coordination can be expressed by means of a covariation measure (an example of a covariation measure in independent data is the well-known covariance, an example of a covariation measure in smoothed time series is the sum of products of the first derivatives of the smoothed curves). Although the details cannot be discussed here, it turns out that the covariation measures of the observed and simulated trajectories are qualitatively similar (for instance, they are characteristically distinct from covariation in uncoordinated series). Quantitatively, however, covariation appears to be higher in the simulated than in the observed series.

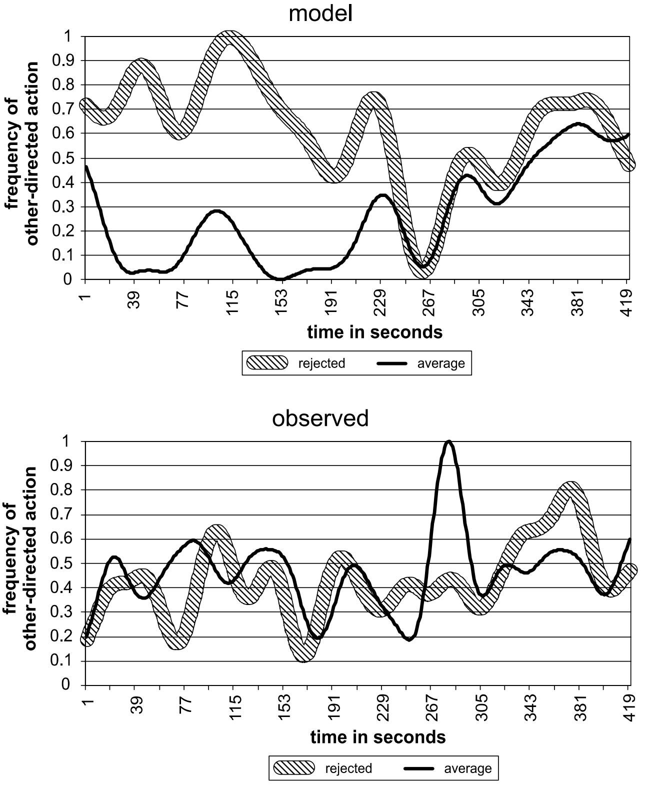


Fig. 5. Examples of smoothed model outputs and data of frequency of other-directed action over time in a rejected dyad.

A second major qualitative property of real interaction is its cyclical nature, i.e., the frequency of interaction oscillates over the course of the interaction event (Newtson, 1993, 1994; Warner, 1992). That is, the frequency and/or intensity of other-directed action goes up and down, resembling a landscape of rounded hills. If the observed data and the simulated data are smoothed with the same smoothing parameters, the number of hilltops found in the observed and simulated data should be comparable.

Hilltops can be found by means of peak fitting software that estimates the minimum number of peaks required to describe the form of the interaction curves (the software used is Peakfit 4.12; note that this peak fitting procedure differs from the more frequently used Fourier and spectral analysis which could also have been used here; Warner, 1998). The number of peaks varies between 6 and 10, although it appears that the simulated trajectories show on average less peaks than the observed ones. Peaks in the other-directed action of the child and the play partner tend to be coherent with one another in terms of their position on the time line and their duration (but they often do not correspond in terms of amplitude). Correspondence between peaks—which refers to the level of mutuality or coherence in the interaction, and which is comparable to the covariation measures described earlier—can be expressed in the form of a correlation matrix of the peaks. These correlation matrices are highly similar for the observed and the simulated data and thus demonstrate an important qualitative correspondence between real interaction and the model.

While discussing the qualitative similarities, we just noticed some quantitative dissimilarities between simulated and observed trajectories, such as the number of peaks or the magnitude of the covariation. Another difference is that the simulated trajectories sometimes become trapped in a particular behavioral mode, such as continuous other-directed action or continuous self-directed action. It is likely that all these differences are due to the fact that the model captures idealized, prototypical types of dyads, whereas the real dyads are more variable and less extreme in their properties.

## *Short- and long-term dynamics: Connecting the model of social interaction with a model of social development*

### *Long-term dynamics as a chain of short-term dynamics events*

The short-term dynamics of a play session or other form of interaction that involves dyadic interactions can be conceived of as a single step in a time series of such interactions over a long term, for instance a school year, or a developmental period such as childhood. The long-term development concerns the long-term change of the parameter values that govern the short-term model. For instance, it is possible that in a child who encounters many unsuccessful interactions with others and who thus cannot sufficiently realize his desire (or "concern") for pleasurable interactions with others, a slow decrease of the concern parameter will occur, leading to diminishing interest in others and thus to a more or less self-caused rejected status. To build a long-term model, i.e., a model of connected short-term interactions, a number of additional parameters or variables must be specified, which are implicit in the short-term model. An important example of such a variable concerns the child's choice of an interaction partner and thus requires the availability of a range of potential partners (e.g., the child's class mates, children from the neighborhood, etc.). Choosing an interaction partner simply means to initialize a social action directed towards a specific child from one's social environment. If the initialization is responded to by the other child, an interaction emerges that is more or less satisfying for the child(ren) in question, depending on the course of the interaction, the children's interests, skills, etc.

To account for the short-term selection of interaction partners, the long-term model must allow for some sort of representation in the child of the child's network in terms of past experiences of how pleasurable the interaction with a particular member from the child's network actually is. This is not a representation in the mentalist sense, which the Bloomington approach would reject. It is most likely some sort of neural network of fuzzy emotional values associated with memory traces related to the members of the child's social network, which can eventually be activated by particular information from the environment.[3]

The child will tend to select those interaction partners with whom a pleasurable interaction can be established. It is likely that such continuous selection and resulting interactions lead to a differentiation in the social network, in terms of friends (with whom the child attempts to engage frequently) and others

However—and this point is consistent with the concept of social status—the child is not only selecting others, others are also selecting the child (and it is this factor that makes the

[3] Which on the adult level is nicely illustrated by a line from the famous Dutch singer, Corry Konings, "Hoe-ze-heette-dat-ben-ik-vergete-maar-haar-kussen-vergeet-ik-nooit- meer," that is, "how-she-was-called-I-have-forgotten-but-her-kisses-I'll-never-forget."

difference between rejected and popular children, for instance). If another child attempts to interact, the child may either respond to it or not, and thus establish a short-term interaction with the other child (or not), even if the other is not the child's own choice. It is this dynamics of selecting and being selected by interaction partners that leads to changes in the short-term model parameters and that thus can simulate the long-term emergence of friendships or of becoming a popular, average of rejected child in a group (Ballato, van Geert, & Bosma, 2005).

*Links between the short- and long-term parameters*

Although the long-term processes—such as friendship formation or social isolation—are of a different nature than the short-term processes of interaction and emotional expression, they can be expressed in terms of the parameters and variables that govern the short-term process. For instance, friendship means a high preference for interaction with another child who has a high preference for interacting with the first. In the short-term model, this preference is simply the initial level of the concern parameter ("how pleasurable is the interaction with the other child") in both children. In this way, the long-term model consists of the iterative succession of many dyadic interactions, with a new component added, namely a social network of potential interaction partners. By iterative succession we mean that the properties of a particular process of dyadic interaction at time $t$ affect the parameter values of the successive dyadic interaction at time $t + n$ (with $n$ referring to a considerably longer time span than the "1" that features in the short-term model). The long-term model must of course specify how a preceding interaction affects a successive interaction. For instance, how is the preference-parameter that is associated with a potential interaction partner, updated after an interaction with that child in question, and how does this updating depend on how pleasurable the past interaction was. A further discussion of these update functions are beyond the scope of the present article. The most important point here is that a short-term model is in fact a nested component of a long-term model.

In building dynamic systems models of development, it is not strictly necessary to link short-term dynamics models to the long-term-dynamics, as was illustrated in the present section. It is possible to confine oneself to models of the long-term process without specifying which short-term dynamics are required. An example of a dynamic model that focused on the long-term dynamics only is the growth model of vocabulary discussed earlier. A potential short-term dynamic model of vocabulary might consist of a dynamic model of word learning in the context of communication between a child and a parent (an example of such a model focusing on the dynamics of word learning can be found in Colunga & Smith, 2005; Jones & Smith, 2005; Samuelson & Smith, 2005; Smith, 2005; Yoshida & Smith, 2005). It is highly likely that the growth parameter in the long-term vocabulary model can be decomposed into the short-term dynamics of exposure to and imitation of new words, for instance. It goes without saying that although the coupling of a long- with a short-term dynamics model is not strictly necessary, it nevertheless provides a deeper and more complete explanation of the developmental process at issue.

## Conclusion and discussion

A major goal of this article was to show that what we have—somewhat unscientifically—called the Groningen brand of dynamic systems might contribute to the understanding of developmental processes. The main idea of our approach was cast in the form of an elementary dynamic equation, which was compared with the elementary equation that underlies the majority of developmental research (which we called the standard approach). The latter equation represents an association between an independent variable on the one hand (e.g., maternal word use) and a dependent variable on the other (e.g., child vocabulary growth). The dynamics equation represents an association between an earlier state of a variable (e.g., vocabulary today) and a later state of the variable (e.g., vocabulary a week later). The basic notion of a self-iterating process that lies behind this equation is characteristic of dynamic systems approaches in general.

The dynamic approach has the following contributions to make.

*Adding a process dimension to existing research*

In this article, we have tried to show that these basic equations represent different approaches to the same phenomenon—vocabulary growth, or social development for that matter—which can best be seen as complementary. That is, a combination of a dynamic with a standard approach leads to a deeper understanding of the process and mechanisms of development (see Lewis, 2004 for a comparable conclusion). In standard models, assumptions about the mechanism or process of change are most often implicit. For instance, if one shows that a high diversity of maternal words corresponds with a higher level of vocabulary development in the child, the implicit mechanism is that exposure to many diverse words increases the growth of vocabulary. A dynamic systems model of the vocabulary growth phenomenon will try to obtain a more explicit representation of this mechanism and by doing so often introduces additional aspects, such as the potential effect of the child's vocabulary on the mother's diversity of expression, or the potential effect of vocabulary on grammar in the child (and vice versa). The dynamic model generates specific testable hypotheses that often go beyond those based on the more standard models and thus contribute to a deeper understanding of the developmental process.

A side issue relates to the question what a dynamic systems approach might add to the now so successful brain and genetic explanations of development. The answer is that genetic or brain-based explanations are often of the $y_i = f(x_i)$-type and do not necessarily provide a process explanation. Given that genes contribute to a considerable extent to a certain developmental outcome is important knowledge, but it implies that a process-oriented question should be asked, namely how is it that genes operate in the process that leads to the outcome at issue (e.g., how do genes relate to reading disabilities in children; Plomin & Kovas, 2005). On the brain side, Fischer and colleagues have argued that brain and behavioural development are dynamically linked and that dynamic systems models can play a significant role in furthering the understanding of basic developmental processes (Dawson & Fischer, 1994; Fischer & Rose, 1994).

*Methodological consequences*

We have argued that dynamic models not only complement standard independent–dependent variable models, but also that they provide a distinct view on measurement and on the link between short- and long-term processes of change. As to the first aspect, the dynamic approach requires the availability of developmental rulers, i.e., measuring instruments, tests and so forth, that are able to capture the entire developmental process, and not just a relatively small fragment of it. Applying a dynamic systems view thus obliges the researcher to construct large-scale measurement instruments, i.e., instruments covering a long time span and a multitude of contexts, that are able to put developmental phenomena into a broader framework (Fischer & Dawson, 2002). In addition, the dynamic systems approach—and this includes all different approaches to dynamic systems theorizing in developmental psychology—has emphasized the importance of phenomena such as intra-individual variability or fluctuation, vagueness and fuzziness. They were traditionally seen as forms of measurement error, but in reality, they can provide relevant information about the underlying developmental processes. Finally, the dynamic systems approach argues for time-series designs, i.e., series of observations in single subjects that are frequent enough to capture the relevant properties of the underlying developmental process. In summary, the indirect methodological contribution of the dynamic systems approach to developmental psychology is that it has enriched the sources of information that researchers may draw on.

In addition to the issue of measurement, a second methodological consequence of the dynamic systems approach concerns model building and mathematical model building in particular. To begin with, model building is not an obligatory aspect of a dynamic approach. It is reassuring to know that, in view of the mathematical complexities that dynamic systems theory may entail, meaningful dynamic systems contributions to understanding development can be made without any reference to equations. On the other hand, the aim of developmental research is to understand development, not to understand mathematics. Thus, the function of a mathematical model, whatever its nature, is to express the sometimes complex relationships in a concise way and to provide a means for inferring predictions or any other type of conceptual consequence in a more rigorous way. Even simple equations may thus contribute to a further understanding of a particular process. One of their important functions is that they oblige the researcher to be as explicit as possible about the theoretical assumptions needed to construct a usable model.

## *Back to Bloomington…*

The last section of this article refers to the approach of the person to whom this special issue pays tribute, Esther Thelen. For Thelen, dynamic systems theory is a specified theory of development, putting the embodied, physically acting person to the fore, defining behavior, learning and development as an activity that takes place in the continuous loop between an acting person and a specific environment. In this perspective, understanding development means to understand the chain of causes and consequences that leads from an earlier state (e.g., birth) to a later state (e.g., the end of childhood; see Smith, 1999). The authors of the current article share this notion of the centrality of causal chains by emphasizing the $y_{t+1} = f\ (y_t)$-structure of any theory of development. They take a somewhat different stance with regard to the first point, the embodied agent, and assume that dynamic systems models can be formulated for anything that can be conceived of as a dimension or variable, such as vocabulary, theory-of-mind or social preferences. The use of these particular terms implies no ontological claims with regard to the nature of those phenomena, for instance, it is not implied that theory-of-mind is some sort of mental structure. For all that we know, it is a particular way of patterning perceptions, actions and expectations regarding the behaviour of other people. However, at the end of the day, these variables and dimensions will have to be brought back to the working of a so far unknown short-term dynamics that incorporates the embodied acting person that Esther Thelen brought in to the study of human development.

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